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DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

CONSTRUCTION AND TESTING OF HYDRAULIC MODELS, MUSKINGUM WATER-SHED PROJECT

By George E. Barnes[1], M. Am. Soc. C. E., and
J. G. Jobes[2], Jun. Am. Soc. C. E.

Synopsis

Hydraulic model studies were conducted on eleven of the dams for the Muskingum Water-Shed Project, north of Zanesville, Ohio. This paper describes the laboratory facilities used in the tests, and the more important features of model fabrication, operation, and test procedure. The time and personnel required, and the cost of doing the work are given. An effort is made to present a picture of the physical equipment and organization required, and also the cost of securing results on a well defined, although not necessarily typical, program of model studies. It is believed that such material, as distinct from technical findings alone, has not formerly been made available to the extent desired by the profession. Thus, more discussion is devoted to the manner in which the work was done, than to a detailed analysis of the test measurements. However, certain selected test observations of special importance, which are considered to be of general interest as well, are presented herein.

Method of Attack

In each of the eleven studies, a working model simulating the outlet works, spillway, or other features governing hydraulic performance, was constructed from preliminary drawings of the dam. The general purpose of all tests was: (1) To observe and record the behavior of the model, built in accordance with the preliminary design; (2) to determine revisions necessary or desirable for optimum performance; and (3) to revise the model to complete or substantial agreement with the final design and to make complete record tests under the full range of flow conditions anticipated for the prototype.

Note.—Discussion on this paper will be closed in April, 1937, *Proceedings*.

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[2] Associate Engr., Los Angeles Office, U. S. Engr. Dept., Los Angeles, Calif.

The laboratory personnel was given the relation between tail-water stage and discharge at each site, based upon channel studies. In certain cases, where considerable variation in stage might be expected at a given discharge, the best performance was demanded only for most frequent operating conditions, good performance for less frequent circumstances, and safe operation under circumstances likely to occur only at rare intervals. Adequate discharge capacity and safety were of first importance.

The models were constructed with surfaces in contact with the water (except non-essential detail) similar to the proposed structure. The models were not distorted, and were built to the same vertical and horizontal scale, varying from 1:20 to 1:40. Each model included some or all of the following elements: (*a*) Reservoir approach topography at the control works; (*b*) intake tower and gates; (*c*) outlet conduits or tunnels; (*d*) stilling-basin; (*e*) outlet channel; and (*f*) spillway.

The laboratory studies extended from August 1, 1934, to June 20, 1935. They began immediately after the preparation of the official "Plan of Flood Control and Water Conservation Reservoirs" by the United States Engineer Office of the Zanesville District, and were continued throughout the design period. As the basic data for design were supplemented by field measurements and revised by analysis, and as laboratory test data accumulated, there was a constant rotation of data between the office and the laboratory during this interval. When acceptable performance for stipulated operating conditions was secured in the model, record drawings of the final model were put in the hands of the designers. These drawings were supplemented by a final report giving a complete record of all tests for that model.

Laboratory Facilities

The model studies were made in the Warner Hydraulic Laboratory of the Department of Civil Engineering, Case School of Applied Science, in Cleveland, Ohio. This laboratory includes two flumes, both of which were used in the work, and the larger one of which was added specifically for these tests. Fig. 1 is a longitudinal elevation of the circulating system, showing the metering devices and the flumes with essential accessories.

The laboratory occupies a total of 6 000 sq ft of floor space on two floors of a modern building, with a trucking entrance on the ground floor and a 3-ton traveling crane for equipment and material handling. From a 50 000-gal recirculating reservoir under the ground floor, a battery of three centrifugal pumps, operating in parallel, with a total capacity of about 7.5 cu ft per sec, deliver through a 10-in. common header to a constant-level tank on the second floor, or to the supply mains direct, if desired. Flow from the constant-level tank is through a 12-in. or a 4-in. supply line, each equipped with a Venturi tube, the flow being registered with indicating, recording, and totalizing meters. The metered supply reaches the River Flume on the ground floor through a 10-in. riser, the discharge from the flume being returned to the recirculating reservoir. On the second floor, the Experimental Flume is fed by a 10-in. take-off from the riser, or from a 6-in. take-off and weir box.

Discharge from the experimental flume on the second floor reaches the reservoir through a 10-in. return, or may be passed through any one of six volumetric measuring tanks, each 6.0 ft in diameter and 10.0 ft deep, arranged

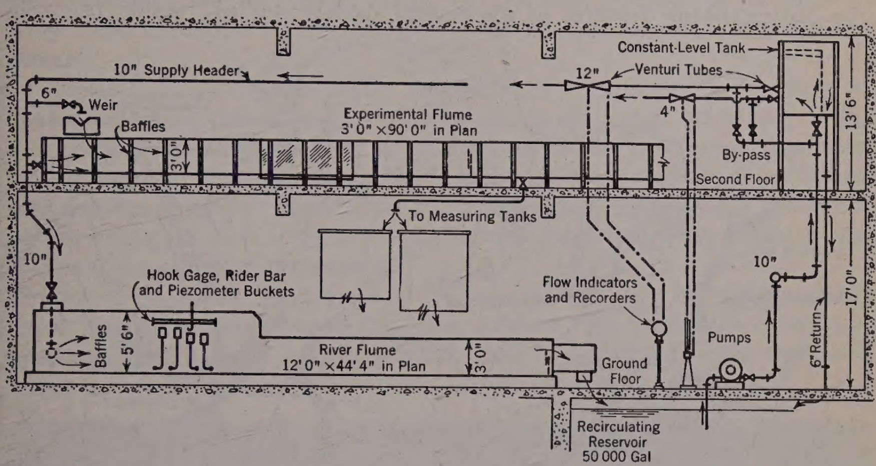


FIG. 1.—LONGITUDINAL ELEVATION OF HYDRAULIC LABORATORY (NOT TO SCALE)

in pairs, with flipper devices for continuous running to calibrate the meters or measure the discharge, and each equipped with telescopic level readers and quick-opening valves through which water is returned to the reservoir.

Both the second-floor experimental flume, with glass section, and the river flume on the ground floor, are equipped with adjustable tail-gates and with numerous piezometer connections. Each separate model requires a newly built piezometer assembly, leading in each case to a battery of open buckets in which levels are read by a hook-gage traveling on a carefully leveled rider bar. Each separate model, likewise, requires newly set rails supporting point-gage bridges, or sounding devices for measuring scour.

Required flows are set by valve manipulation and indicated by either the 4-in. or the 12-in. Venturi tubes. At low flows, a U-tube manometer containing carbon tetrachloride is used in tests as it gives much greater deflections and greater precision in reading. To insure minimum error, the 4-in. and the 12-in. meters are rated by volumetric measurement. The models for the Wills Creek, Charles Mill, Mohawk, Dover, Mohicanville, and Pleasant Hill Dams (see Fig. 2) were tested on the ground floor and the models for the Tappan, Clendening, Piedmont, Senecaville, and Bolivar Dams were tested on the second floor.

Any description of laboratory facilities needed for model studies would be decidedly incomplete without mention of the shop facilities, tools for maintenance work, and the fabrication of special instruments, or other devices required for a wide range of purposes. In a large city such as Cleveland, the availability of industrial shops is an added asset. Furthermore, room is needed for the preparation of drawings, filing of records, and reference material.

Types of Models

The characteristics of the various dams have been described by E. L. Winslow, Jr., Assoc. M. Am. Soc. C. E.[3] The following description gives only the distinguishing features of the several models studied. Aside from site

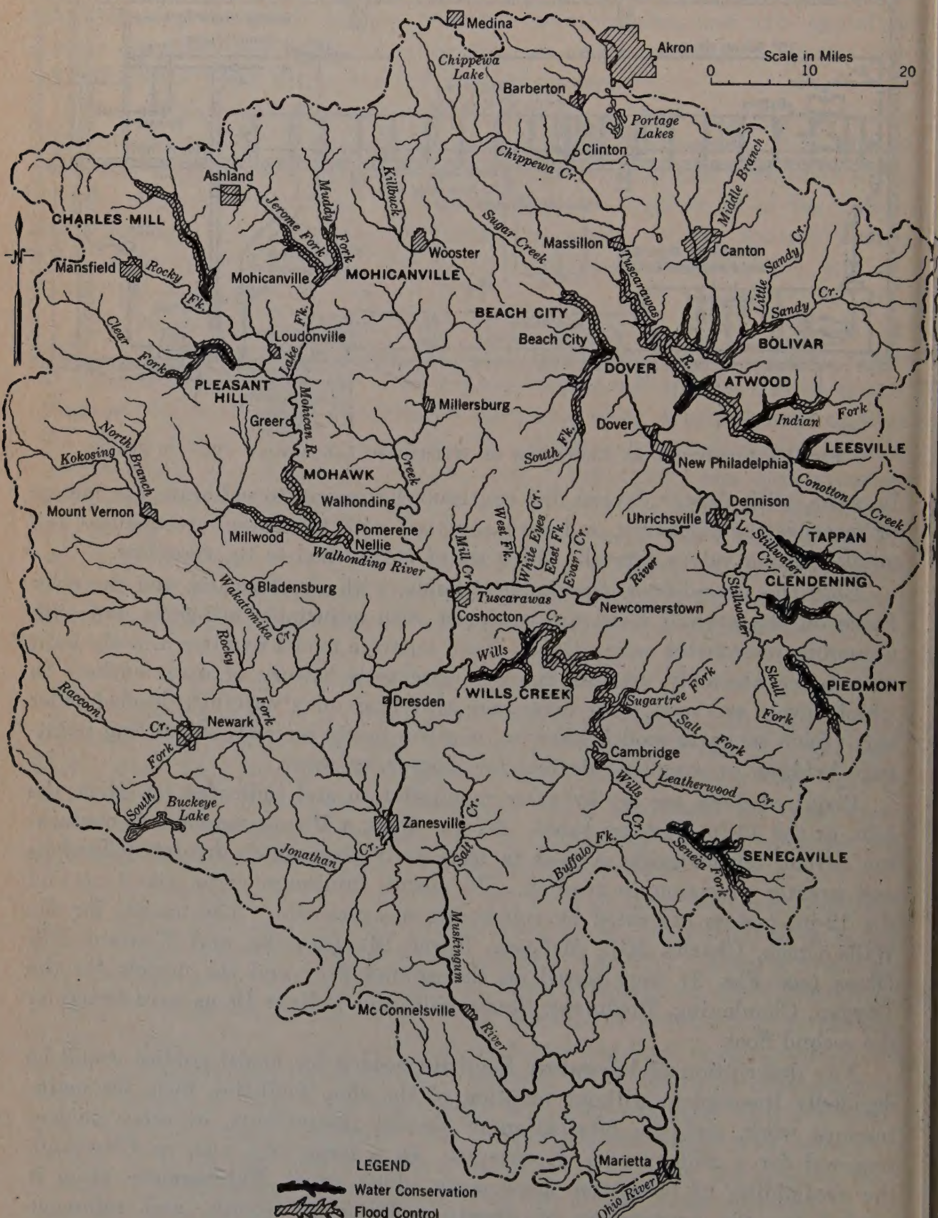


Fig. 2.—Official Plan of Reservoirs and Dams in the Muskingum Water-Shed in Ohio.

[3] *Civil Engineering*, January, 1936, p. 1.

requirements, dissimilarity between structures is occasioned partly by the fact that operation of the several works was planned so as to be interrelated, and the performance of one outlet, in certain cases, affects another. These circumstances result in a great variety of hydraulic problems to be solved in design, and the model studies, therefore, vary in character to a commensurate extent.

For each dam, the outlet works terminates in a stilling-basin in which an hydraulic jump is formed, to dissipate the energy of conduit discharge. Since, in any case, spillways on these dams discharge under emergency conditions only to safeguard embankments, model spillways were studied in the laboratory only for those few dams where the confluence of spillway and outlet discharge occurs in such a manner as to affect, appreciably, the design of the spillway, the outlet works, or both.

The Wills Creek, Mohawk, and Bolivar models each include an intake tower with six control gates discharging through twin tunnels or conduits of horseshoe section, into a stilling-basin. The dimensions, heads, capacities, and tail-water conditions differ widely for the three dams.

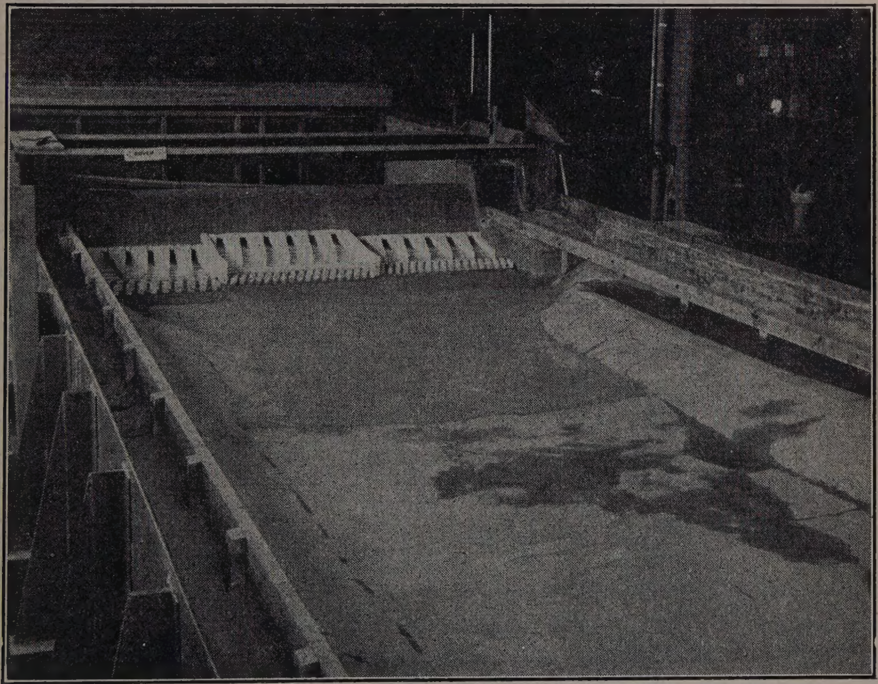


Fig. 3.—Model of Spillway and Outlet Works, Dover Dam

The Clendening, Tappan, and Piedmont models each discharge through a single horseshoe conduit into a different type of stilling-basin. The Clendening, Tappan, and Piedmont Basins are similar to each other, but unlike those developed for other dams of the project. The distinctive feature is the rise of the curved apron below the portal to an elevation higher than the invert

of the tunnel, followed by a parabolic drop to the floor of the stilling-basin. The formation of an hydraulic jump with relatively high tail-water and low tunnel elevation and discharge is thereby obtained.

The Charles Mill and Mohicanville models, as finally crystallized after substantial changes to the preliminary spillway layout, include short conduits through a solid masonry section with a secondary or emergency spillway immediately adjacent. The Senecaville model discharges through twin short conduits set in a gate-tower centered between adjacent twin over-flow sections, each surmounted by a Taintor gate. For Senecaville, the discharge from all outlets combines in a long paved channel with a steep grade leading to a stilling-basin, 400 ft (20 ft in the model) down stream.

The model of Dover Dam (see Fig. 3), simulates a solid masonry, overflow section, 338 ft long (9.66 ft in the model), between abutments, with eighteen gate-controlled, short, outlet conduits through it. These conduits are set in three batteries of six units each, at three levels. The outlet portals discharge over a curved apron between training walls into a stilling-pool which is required to be very shallow because of foundation conditions at the site.

The model of Pleasant Hill Dam (see Fig. 4), includes an intake tower, enclosing a vertical shaft, into which flood waters are discharged through six

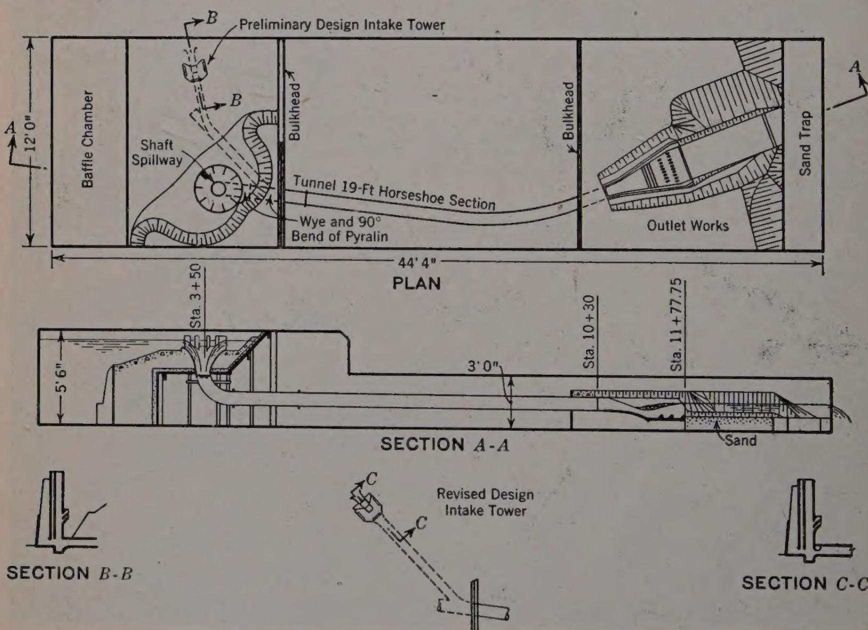


FIG. 4.—MODEL LAYOUT, PLEASANT HILL DAM

orifices, without control gates. The orifices are set at two different elevations, and are dimensioned to secure capacity which will predetermine the reservoir rise automatically. The model shaft discharges at the base through a constricted throat (for the purpose of maintaining a plunge pool in the shaft)

into a 6-in. (15-ft on a 1:30 scale), circular tunnel, and blow-off gates are provided at the tower base, as well as temporary openings for diverting water during the construction period. The spillway is of the shaft type with a circular crest, 2 ft 8 in. (80 ft) in diameter, surmounted by piers to eliminate vortex action, and with its ogee surface converging to an 8-in (20-ft) circular shaft and 90° bend to the horizontal. The model tower and spillway tunnels meet in a wye junction to form a tunnel (horseshoe section) 8 in. (20 ft) in height and width, leading to a stilling-basin below the dam.

Model Fabrication

From the dimensions on the design drawings the Laboratory Staff prepared detailed model drawings for the fabrication of the several elements, and also layout sheets to guide assembly and erection. In making the models, skilled labor was employed in the laboratory and shop. Certain units were made for a price by commercial organizations in Cleveland. In the special instance of the wye-tunnel junction for Pleasant Hill (required to be made of a transparent material so that flow inside the junction could be observed and photographed), the piece was fabricated at the University of Iowa, at Iowa City, Iowa, from drawings prepared at Case School, and then shipped to Cleveland for erection. Speed was essential and all available facilities were needed. Where complex details had to be reduced to model dimensions, as, for example, in the gate-towers, and tunnel and portal sections, the model drawings were made to full model scale. Topography in the model reservoir and in the channel below the dam was reproduced in the model by inundated sand fill, topped with a $1\frac{1}{2}$-in. mortar coat, placed between sheet metal templets cut to the proper transverse profile and set at several stations on the center line of the works. A tinsmith cut these templets from full-sized paper patterns. The original pencil fabrication and layout drawings were usually handled by the craftsmen working on the model, and due to the different scales required for the drawings, it was not always feasible to establish a uniform size for the sheets. Experience indicates that it might be preferable to maintain a uniform size for filing and to work from blue-prints, since the drawings suffered considerably in the handling.

To secure the complicated shapes with reasonable accuracy and speed, it was found best to work in sheet steel. The model tunnels and gate-towers, therefore, were made from welded 16-gage material. The Morning Glory spillway was spun from sheet aluminum, on a laminated solid wooden core, accurately turned to shape. The curvature in the tunnels was made as a series of short tangents. The joints between tunnel sections were male and female with bolted flanges. In forming the multiple short conduits—as for the Dover, Charles Mill, and Mohicanville Dams—a box-like steel shell formed the floor, ceiling, and the side walls of the outermost conduits. Individual conduits were then formed by wood-block inserts or piers fashioned by a pattern-maker with snug fit between floor and ceiling. The gates for all dams were vented immediately down stream. These vents were reproduced accurately in the models with welded steel, with openings carried through risers, headers, or

stacks as in the prototype, either to scale or slightly larger. The stilling-basins were usually formed with wood floor, walls, baffle-piers, and terminal sill. Where the side walls were warped, it was easier to secure proper dimensions with 22-gage galvanized sheet metal, or some comparable commercial product. Where changes were not anticipated the walls and floor of the stilling-basin were formed in mortar.

Numerous piezometer connections were installed along the floors of the conduits and at other points where pressure readings were desired. Unless piezometer openings are mechanically true, and unless the surfaces in which they are set are absolutely smooth and undistorted, false readings will be secured. For these models the best piezometer connection was found to be made by brazing a solid lug about 1 in. long to the under side of the steel floors, and drilling either a single $\frac{1}{8}$-in. hole through it in a direction normal to the surface, or three of such holes, and dressing the opening at the surface

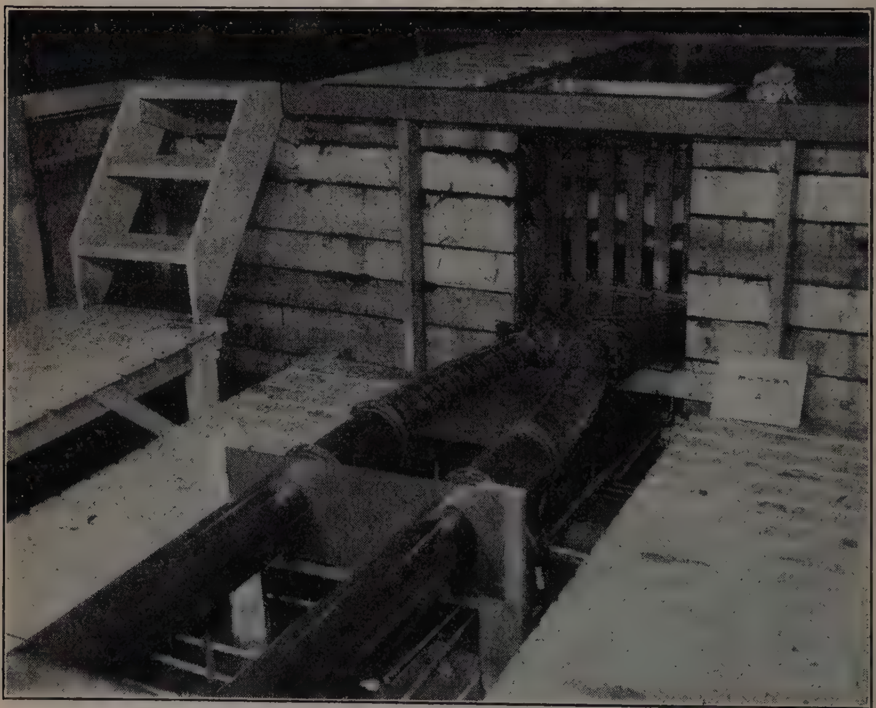


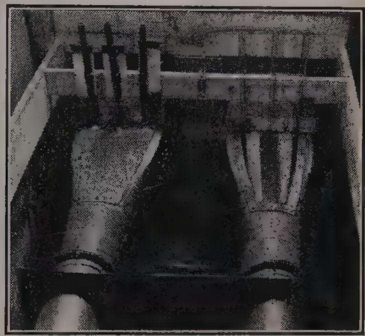
FIG. 5.—MODEL OF INTAKE AND TWIN TUNNELS, MOHAWK DAM (ORIGINAL DESIGN BEFORE REVISIONS)

with a slight radius. The opposite end of the lug was tapped for the insertion of a $\frac{1}{4}$-in. or a $\frac{3}{8}$-in. pipe leading to the piezometer buckets. On the surface of the Morning Glory spillway of the Pleasant Hill model seven pairs of piezometer openings, diametrically opposed, were connected through similar lugs to copper tubing leading to a piezometer panel on which was mounted 0.5-in. transparent tube through which the pressure gradient was read directly against paper scales. It should be remembered that piezometers record the

pressure and not the water depth where curvilinear flow exists in a vertical plane. Of all the measurements taken, piezometer readings were most difficult and unsatisfactory.

Fig. 5 shows a model of sheet steel with the intake ceiling elliptical in profile and of transparent material. The gates are steel flats, sliding in

(*a*) TWIN INTAKES



(*b*) GATE STRUCTURES

FIG. 6.—TYPICAL MODEL CONSTRUCTION, BOLIVAR DAM

grooves in the wood division piers, and vented through rectangular slots and stacks. The tunnels are 20 ft high (model scale, 1: 40). The complete model for the Dover Dam to a scale of 1: 35 is shown in Fig. 3. Typical model construction is shown in Fig. 6[4]. The Bolivar tunnels are 16 ft high to a scale of 1: 40. Typical of the stilling-basins for Clendening, Piedmont, and Tappan, is the model shown in Fig. 7.

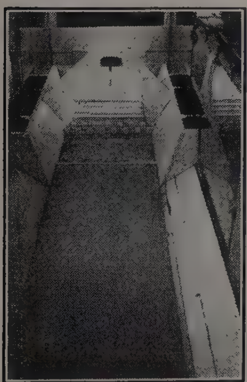


FIG. 7.—MODEL OF STILLING-BASIN, CLENDENING DAM

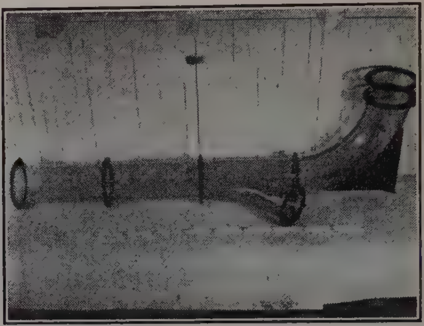
FIG. 8.—MODEL OF INTAKE TOWER, PLEASANT HILL DAM

Some of the most complex model fabrication was required in the case of the Pleasant Hill Dam. Fig. 8 shows the model intake tower of welded steel sheet. The Morning Glory spillway of spun aluminum with piezometer

[4] See, also, *Civil Engineering*, January, 1936, p. 16.

connections is shown in Fig. 9(*a*). The wye transparent junction and steel horseshoe tunnel are exposed for their full length to provide access to piezometers. The wye is shown in Fig. 9(*b*).

(*a*) SPILLWAY



(*b*) TUNNEL JUNCTION

FIG. 9.—TYPICAL MODEL CONSTRUCTION, PLEASANT HILL DAM

MEASUREMENTS

For each of the eleven dams certain features of operation and performance were studied, including:

(*a*) The relation of reservoir stage to discharge, for various combinations of outlet openings;

(*b*) The distribution of energy losses;

(*c*) The capacity and behavior of the spillways; and,

(*d*) The performance of the stilling-basin, with particular reference to the variation in tail-water level at a given discharge, or the unsymmetrical discharge from the multiple outlets.

The test measurements, therefore, were aimed to secure the following data:

(1) Rating curves of reservoir stage against discharge for all combinations of outlets open or partly open (where tail-water variation affected the rate of discharge, rating curves were secured for the range of tail-water levels);

(2) Water surface profiles through the stilling-basin, extending from the outlet portal through the hydraulic jump to the channel below;

(3) Velocity traverses showing the distribution of velocities across various sections in the stilling-basin, and in the channel below;

(4) Scour (with the sand bed of the outlet channel screeded level before a test run, cumulative scour over an hour's time was measured to compare tests, one with the other);

(5) Flow conditions, recorded by photographs and motion pictures;

(6) Observations without measurement (the effect of rising tail-water on the hydraulic-jump performance was recorded as observed by eye for small increments of rise);

(7) Check surveys to determine the actual measurements of the models at various times as against the measurements to which they were set (in some

cases swelling wood or settlement of supports made it necessary to re-set the model; the actual dimensions of the model tunnels and outlets were recorded); and, finally,

(8) Record tests.

The latter (Item (8)) were so-called formal tests conducted with all the measurements required to determine the behavior under stipulated operating conditions. They included hydraulic gradients through the structure, the upper pool elevation and discharge, the water-surface profile of the hydraulic jump, the distribution of velocities in the stilling-basin, and the topography of scour at the end of an hour's run.

Model Operation

For taking measurements it was found convenient to furnish the laboratory operators with mimeographed sheets, there being a separate form for the: Rating curve; water-surface profile; velocity traverse; hydraulic gradients; scour sheets; and daily diary.

In operation, two men could usually handle the model on an 8-hr shift, one acting as reader and the other as recorder. When the model was being built or dismantled, skilled and unskilled labor was employed in addition. The laboratory operated continuously on a 24-hr, three-shift basis for about seven months. This continuous operation and accumulation of data kept about ten draftsmen busy, on an 8-hr basis, plotting, computing, and recording all measurements and model details, and preparing fabrication drawings for new models.

For a record test, the sand bed below the stilling-basin was first screeded level to the desired contour. From a 2-in. flood line, water was admitted slowly at the tail-gate to approximate tail-water depth for the intended discharge, so that the initial scour during the run would be slight. The proper flow was set by the meters, and the reservoir level and tail-water level were brought to equilibrium. All piezometers were blown free of air and stabilized. The hydraulic gradients were then recorded, water-surface profiles were taken in the stilling-basin with a point-gage and the distribution of velocities was recorded by Pitot tube or Bentzel tube in the stilling-basin and below the terminal sill in the outlet channel. At the end of an hour's run, the flow was stopped, the reservoir was allowed to empty, and the lower pool was drained to expose the sand bed, which was then measured with a sounding rod to record the depth and location of scour.

Scour measurements were taken as an index by which performance between tests could be compared roughly, since it denoted concentration of high velocities or persistent eddies and whorls along the bottom of the channel. Excessive or irregular scour in any case showed poor distribution of terminal velocities. From velocities at two-tenths and eight-tenths of the depth, the discharge was computed by ordinary stream-gaging methods to check by agreement with Venturi meter readings.

For Wills Creek, a preliminary model to a scale of 1:40 was superseded by a final model to a scale of 1:25. For Charles Mill, a large scale (1:20)

model was used for conduit and stilling-basin studies only, followed by a 1:30 scale model complete with spillways.

Time, Personnel, and Cost Requirements

Fig. 10, "Laboratory Schedule", shows the dates between which model erection and model testing occurred. The interval shown for erection and testing does not include time for preparing drawings, for the fabrication of model parts, or for handling and digesting test results. As a rule, no time elapsed between clearing the flumes of one model and erecting another. The vertical lines in Fig. 10 indicate the man-hours for that week's payroll, includ-

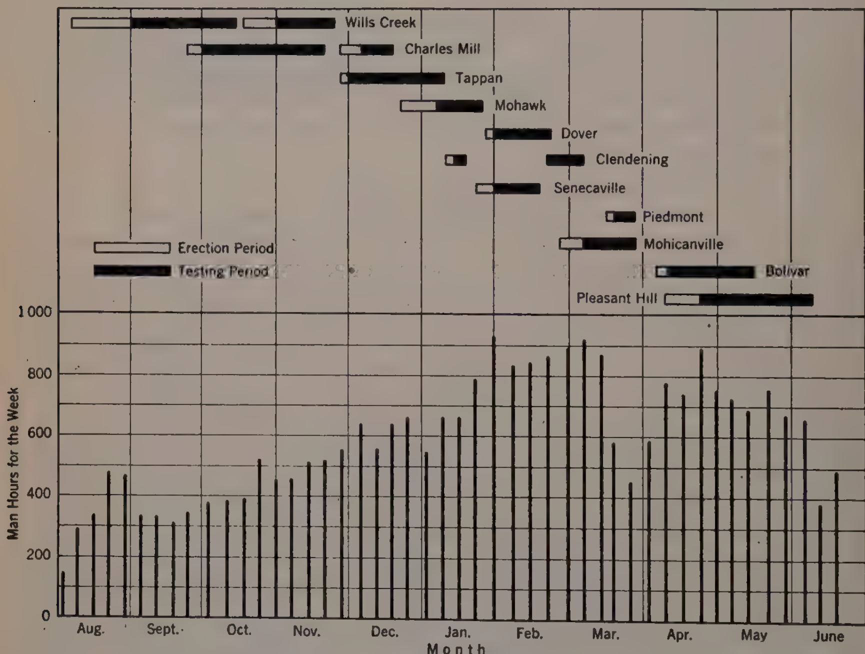


Fig. 10.—Laboratory Schedule, Showing Dates Between Which Model Erection and Model Testing Occurred

ing all operations. Rates for engineering assistants (draftsmen, computers, model builders, and model operators) ran from \$0.60 to \$1.50 per hr, limited to a 40-hr week. Rates for skilled labor (machinists, carpenters, cement finishers, pattern-makers) ran at the prevailing local scale, from \$1.00 to \$1.25 per hr, limited to a 30-hr week.

The direct cost of materials and labor for the several models is shown in Table 1. It includes a total of about \$1 800 for blueprinting required for the several reports and \$1 100 for photographic prints used in the reports. The power consumed in pumping totaled about 20 000 kw-hr. To these data should be added approximately 40% to cover rentals, supervision, miscellaneous services and supplies, travel and consulting expenses, and the salaries of three

engineers from the Zanesville U. S. Engineer Office stationed at the laboratory. Included in the 40% is the cost of the river flume ($1 250), which was installed specifically for these tests and transferred to Case School as part compensation for the work.

TABLE 1.—Direct Labor and Materials Involved in Model Testing

Structure (see Fig. 2)	Scale of model	Units tested	Costs: Labor	Costs: Materials	Costs: Total
Wills Creek....	1:40	Intake, twin tunnels, stilling-basin			
	1:25	Intake, twin tunnels, stilling-basin	$4 650.59	$1 382.48	$6 033.07
Charles Mill...	1:20	Outlet conduits, stilling-basin			
	1:30	Outlet conduits, overflow section, stilling-basin, spillway..........	3 010.00	977.81	3 987.81
Tappan.......	1:20	Tunnel, stilling-basin..........	935.94	277.87	1 213.81
Clendening.....	1:20	Tunnel, stilling-basin..........	972.31	262.87	1 235.18
Piedmont......	1:20	Tunnel, stilling-basin..........	398.03	131.34	529.37
Mohawk.......	1:40	Intake, twin tunnels, stilling-basin.	2 468.06	1 106.39	3 574.45
Senecaville.....	1:20	Outlet conduits, Taintor gate sections, outlet channel, stilling-basin..........	1 106.05	236.46	1 342.51
Dover........	1:35	Spillway section, outlet conduits, stilling-basin	2 038.85	960.52	2 999.37
Bolivar........	1:40	Twin intakes, tunnels and stilling-basin..........	1 231.50	580.02	1 811.52
Mohicanville...	1:25	Outlet conduits, outlet channel, stilling-basin, spillways.........	1 546.56	499.35	2 045.91
Pleasant Hill...	1:30	Intake tower, spillway tunnels, stilling-basin..........	3 167.83	1 574.40	4 742.23
Beach City....	....	Preliminary drawings for model...	33.00	....	33.00
Cost..........	......	....	$21 558.72	$7 989.51	$29 548.23

Exclusive of the building, a 50 000-gal. reservoir, the general plant, shop, and tools, the permanent investment, installed, for the principal items of the fixed equipment used for the tests is:

Three motor-driven pumps and controllers........	$ 3 400
Major piping and valves..........	1 750
Constant level tank..........	300
Venturi tubes and meters..........	600
Experimental flume (3 ft 0 in. by 3 ft 0 in. by 90 ft 0 in.) with drains..........	1 300
River flume (5 ft 6 in. by 12 ft 0 in. by 44 ft 4 in.).	1 250
Measuring tanks	2 400
Total	$11 000

Caution must be used in comparing the cost of model studies, one with the other, because they are likely to be unrelated in character. The costs shown are those for a concentrated extensive program of studies generally similar and related, under a carefully planned and rigorous schedule. Furthermore, for these studies, work ceased the moment major decisions on design could be made, and the models were built in an economical manner for this immediate purpose only.

Operating Difficulties

Mechanical imperfection of the models themselves, together with the physical difficulty of taking precise measurements in all laboratory work, may

seriously affect the results of an hydraulic model study, if these factors are under-rated or their significance ignored. Some of these factors deserving special consideration are: Discharges, piezometer readings, points of filling, orifice coefficients, joints and connections, warping, settlement, water-surface elevations, velocity measurements, and entrained air.

Discharges.—The small geometric scales necessary, require a large ratio of prototype flow to model flow, necessitating the accurate measurement of discharge. Where a weir is used, calibration should be directly by volume or weight if possible. Otherwise, proper precautions must be followed to insure that the weir corresponds in setting to the original by which formulas have been derived. With the Venturi meter, direct reading may be obtained at about full capacity with ordinary differential manometers. As previously noted, it is much more satisfactory with lower flows to magnify the deflections of the manometer by the use of a lighter liquid, such as, carbon tetrachloride. The specific gravity of the carbon tetrachloride must be determined carefully and frequently, and the glass tube should be at least 0.5 in. in diameter for easy cleaning and to avoid the effects of surface tension on the meniscus.

Piezometer Readings.—Where high velocities occur, piezometer readings are the most difficult measurements to obtain most accurately. A smooth $\frac{1}{8}$-in. hole, with a $\frac{1}{16}$-in. radius free of burrs is suggested[5] by C. M. Allen, M. Am. Soc. C. E. The relation between discharge in the prototype and that in the model depends on the ratio, n. Since n must be determined by piezometer readings their importance and the care to be exercised in the fabrication of piezometer connections cannot be over-emphasized.

Points of Filling.—That point at which the model tunnel ceases to flow partly full and begins to flow completely filled, is difficult to determine. Even where this point is determined accurately in the model there may be some question as to whether or not the prototype will behave in exactly the same manner.

Orifice Coefficients.—The use of models in which orifices are involved raises the question as to how the coefficients are affected by actual size. Tests covering small orifices show a variation in the coefficient with the actual size of opening.

Joints and Connections.—As few joints as possible in a model appears to be desirable, inasmuch as these joints affect flow conditions both up stream and down stream with consequent breaks in the hydraulic gradient, as shown in the piezometer readings. A smooth hydraulic gradient is desirable in order to obtain the value of n for the model.

Warping.—It is extremely difficult to prevent warping in wooden models. Slight warping often produces incorrect flow conditions, which affect the results seriously. Many methods of water-proofing have been tried by the different hydraulic laboratories. The use of celluloid dissolved in banana oil, and the use of aluminum foil have been tried.

Settlement.—Settlement of the sand base under mortar surfaces leaves the templets exposed and elevations in error so that often it is necessary to rebuild

[5] "Piezometer Investigation", by C. M. Allen, *Transactions*, A.S.M.E., May 15, 1932.

the model after it has been in use. If the sand base is thoroughly inundated prior to placing the mortar surfaces, and not permitted to dry out, this difficulty is kept at a minimum.

Water-Surface Elevations.—Water-surface elevations are difficult to obtain, and care in leveling the base rail and in checking of the gage zeros, must be maintained at all times.

Velocity Measurements.—Velocity measurements in a model are difficult to obtain inasmuch as they usually cover a wide range, from considerably less than 1 ft per sec to greater than 10 ft per sec. For the low velocities the Bentzel meter, specially constructed Pitot tubes, and small current meters have all been used by the various laboratories.

Entrained Air.—Most hydraulic formulas are based on the assumption that water weighs 62.5 lb per cu ft. However, they do no hold exactly where the hydraulic jump, or where flow conditions in which air is entrained occur. Thus, any model that includes these flow conditions should have a factor of safety added greater than the minimum dimensions shown by the model tests.

Observations and Findings

In model studies, preliminary analysis is essential in order that time and money spent on laboratory manipulation may be minimized. The test measurements themselves are worthy of equally careful analysis and interpretation. Analysis of the detailed test observations, however, is beyond the intended scope of this paper, and the following discussion includes certain findings which are stated in general terms only, to give a brief picture of certain selected results.

The outstanding single result of tests was an improvement to all stilling-basins. In the case of single or twin outlet conduits, the typical features of the model basins are illustrated in Figs. 4 and 7. With the jet issuing from the portals between flaring side-walls over a curved apron, the models demonstrated, for these outlets, that the jump could be held with slight change in location on the apron, for all rates of discharge and corresponding tail-water levels, by block-shaped baffle-piers with stepped up-stream faces, anchored to the basin floor. For a given discharge, the jump appeared to be slightly more stable when the apron itself was stepped.

With excessive flare and batter to the side-walls, lateral distribution of the shooting flow on the apron was poor. In such cases the water failed to follow the flare, and lack of forward momentum at the side-wall permitted back rollers from the tail-water in the lower end of the basin, which skewed the jump to one side. The result was persistence of whorls, eddies, and concentrated high velocities at the exit from the basin. Similar behavior accompanied the presence of excessively high tail-water in any case. A low value of average velocity computed on the basis of high tail-water proved to be a deceptive index to flow conditions, if the jump itself was neither positive nor balanced.

In the models, good results were secured when the side-walls up stream from the jump were not flared more than 1 on 4 (preferably 1 on 5 or 1 on 6),

and battered not more than 1 on 10. The parallel side-walls of the basin, below the jump, were battered as suitable for their construction as paved slopes or retaining walls, as the case might be. With tranquil flow instead of shooting flow, the slopes could be flatter without serious effect on performance, although slopes as flat as 4 on 1 produced eddies in some cases.

For structural or other reasons, the outlet tunnels for several of the dams were originally set lower than proved suitable for good performance with the coincident tail-water conditions corresponding to the several rates of discharge. In some cases, the tunnel was raised as a result of the tests; in others, where this was not feasible, a unique expedient solved the difficulty.

The Clendening stilling-basin has already been specially mentioned. In the original design, the tunnel invert was placed low, and high tail-water in the model drowned out the jump. To secure a good hydraulic jump without raising the tunnel, proper relation between momentum of outflow and tail-water depth was secured by passing the water over a humped apron (as described under the heading, "Types of Models"), through which a narrow gutter was cut for drainage. The hump was first built crudely with wooden flats in the model, and then carefully shaped for easy transition. This feature was also adopted, after trial, in the models for the Tappan and Piedmont Dams as well.

To reduce concentration of scouring velocities along the bed of the outlet channel, a low terminal sill with the up-stream face stepped, was placed at the end of the stilling-basin and was found to be the simplest and most effective, although other types were studied.

One expedient to reduce the length of stilling-basins was to carry the horizontal flare into the tunnel portals. In the first Mohawk model, with portal and tunnel height equal, the portal flare and increase in area resulted in the hydraulic control being shifted from the tunnel section at low flows, to the portal section at maximum flows. With the large portal flowing full at atmospheric pressure, the differential between the portal velocity head and the tunnel velocity head was roughly 20 ft. Consequently, negative head occurred in like amount in the tunnel. When the flare was omitted from the tunnels, and stilling-basin side-walls were adjusted to correspond with increased warp and flare, the hydraulic jump was unbalanced. The flare was restored, therefore, to the portal section, but the portal area (and negative tunnel head) was reduced by dropping the portal crown. Piezometer readings in the model disclosed the shift in the pressure gradient.

Finally, for all stilling-basins, tail-water stages were such that the depth in the basin much below the stream bed was, in general, found to be needless, and the model studies resulted usually in reducing the length or width of the basins, or both. Liberal size alone was found to be insufficient guaranty of adequate performance.

The true head between reservoirs and outlets is difficult to discover. For preliminary designs, outflow was predicated on head measured as reservoir elevation, minus portal invert elevation, minus the average depth at the portal. Actually, the head will be greater, since the jet from the portal is accelerated downward by gravity, and the pressure head on the apron is less

than that caused by an equal depth of still water. The discrepancy between assumption and fact will depend on the sharpness of apron curvature and jet velocity. In the case of the Pleasant Hill model, the true head by piezometers appeared to have a value closer to reservoir elevation, minus portal invert elevation, minus one-half the depth. The model discharge was greater than anticipated and, in fact, substantially exceeded the required capacity, so that the original 20-ft tunnel and shaft was reduced to 19 ft, with incidental changes resulting in reduced cost of construction.

Another feature of the Pleasant Hill model was the transparent wye-junction through which flow conditions, much under discussion during staff conferences, could be observed and evaluated. With certain rates of flow (to occur in Nature at rare intervals), the model showed that discharge from the gate tower would enter the junction in such fashion as to climb the opposite wall and spiral completely around the interior face of the main tunnel.

The Bolivar model was operated with its portal submerged by the tail-water, as would be required for this particular dam at certain outflows, and when the Dover Reservoir, located down stream, is filled to crest level. Quick closure of the model gates caused a sudden inrush of air through the vents, the slug of air traveling down the tunnel to be released at the portal. Release of air was followed by a sudden rush of tail-water into the portal, with severe slap and shock. Disturbance also occurred with the gates stationary in a throttled position, provided the portal was submerged. The gates in the model, of course, can be closed almost instantaneously, whereas the mechanism in the actual structure will not permit any such rapid gate manipulation. Nevertheless, the model showed that it might be important to release the water through a few gates fully opened rather than through several in a throttled position, and to restrict the speed of gate closure.

With multiple short conduits through a masonry section, as for Charles Mill and Dover Dams, increase in capacity or improved lateral spread on the apron was attempted with conduits having a curved or offset ceiling profile, or horizontal flare, or both. The benefits, if any, indicated in the models were not considered commensurate with the probable cost of the more complex shapes and, in each case, the final models crystallized into conduits of simpler uniform rectangular section and liberal elliptical easement at the inlet, which appeared to be entirely suitable.

The size and position of training walls, and the shape of rock-cut benches by which spillway discharge could be directed with a minimum disturbance into the outlet channels, were studied in the Charles Mill and Senecaville models. Studies of alternate types of spillways were made for Mohicanville. The approach cuts and wing-walls at the inlet were improved in the models for Wills Creek, Charles Mill, and other dams.

Relation Between Capacity of Prototype and Model

To check the stipulated capacity of the outlet works at a given reservoir stage, it is necessary to convert the model discharge into corresponding flow in the prototype. Where gravity is the predominating factor in inducing

flow, with comparatively slight retardation from friction, the flow in the model is directly converted to flow in the prototype by the $\frac{5}{2}$ power of the scale ratio. Such a conversion factor would be appropriate for overflow spillways, stilling-basins and (for simplicity, and on the safe side, for capacity), for short conduits. In long tunnels where frictional retardation has a great influence on discharge, it was difficult or impossible with these model scales and structures to secure an interior roughness factor, or Kutter's n, low enough to give the foregoing conversion factor where n in the prototype is to be 0.011 or 0.013. Consequently, it was necessary to determine the value of n for the model by observing, carefully, the hydraulic gradients in a straight section where uniform flow obtained. For example, the value of n for the model of the Mohawk Tunnels (1:40 scale) was found to be 0.0091, which indicates that the models, however smooth mechanically, are nevertheless too rough to simulate conditions in the prototype. However, knowing the value of n for the model, the corresponding flow in the prototype can be computed by Manning's formula, or by other accepted formulas. For these models too much confidence is not warranted in the experimental values of n, because the model tunnels are believed to be too short to establish uniformly good flow conditions.

Summary and Conclusions

The fact that hydraulic models cannot be made to reproduce, in exact miniature, all the phenomena that will occur in the prototype does not detract from their genuine utility. The performance of hydraulic models may be interpreted to great advantage by a designing engineer. It is obvious that certain of the foregoing findings could not be determined over the drafting-board, or by analysis alone, and that other measurements furnish the data for needed analysis. The model studies described in this paper were in the nature of check tests for the sole purpose of expediting the production schedule demanded of the Designing Staff for this project, by contributing a valuable supplementary method for attacking complex problems of hydraulic design. The tests more than saved their own cost by actually reducing construction expenditure, and, in addition, gave invaluable assurance as to the probable behavior of the finished structures of the Muskingum Water-Shed Project. It is hoped that the data presented herein may be helpful in planning other model studies of somewhat related character.

Acknowledgments

The model studies were conducted under the direction of the senior writer, with the junior writer assigned as Assistant in active charge of laboratory operation. W. A. Snyder, Junior Engineer, and L. G. Leach, Jun. Am. Soc. C. E., Junior Computer, were also stationed by the Zanesville Engineer Office at the Laboratory. The Laboratory Staff included G. Brooks Earnest, Assoc. M. Am. Soc. C. E., W. J. Hopkins, and fifteen to twenty Engineering Assistants at the Case School of Applied Science. The co-operation of the Designing Staff throughout the work was most helpful.

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PAPERS

ANALYSIS OF STRESSES IN SUBAQUEOUS TUNNEL TUBES

By Anatol A. Eremin,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Stresses in a concrete tunnel tube with flexible horizontal ties, are analyzed in this paper. The equations for computing stresses in the tube and ties are based on the Maxwell theorem of reciprocal displacements. Stresses in a tube may also be determined by the Method of Least Work, although the resulting equations are lengthy and require considerable computation. The dimensions of a tube and its tie-rods are generally proportioned by trial and error and, therefore, it is essential to save time in the analysis of the stresses. Changes in the dimensions of the tube and ties require the computation of fewer terms in the equations based on the Maxwell theorem than in the Method of Least Work.

Introduction

The elastic tube lining with flexible horizontal ties has found wide application in the construction of subaqueous tunnels. In Fig. 1, for example, the roadway is placed at the center of the section. The upper horizontal ties are enclosed in the ceiling slab, the space above being used for an exhaust ventilating duct. The lower horizontal ties are placed in the roadway slab, the space beneath it being occupied by the fresh-air ventilating duct. These rods cause a more even distribution of the stresses in the ring.

The tunnel lining may be either a reinforced concrete tube or a series of cast-iron rings bolted at the flanges. Each type has been found advantageous from a construction or economic standpoint, depending on the site and the soil to be penetrated. In the Posey Tube under the estuary channel between Oakland and Alameda, Calif., reinforced concrete pre-cast sections were used. In this case the horizontal steel tie-rods were placed in the ceiling, the roadway slab being designed to act as a strut to resist external soil pressure.[2] In

Note.—Discussion on this paper will be closed in April, 1937, *Proceedings*.

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[2] *Engineering News-Record*, October, 1924, p. 720.

the Holland Vehicular Tunnel under the Hudson River between New Jersey and New York, the tube was made of bolted cast-iron rings. The high tensile steel tie-rods were placed in the ceiling and roadway slabs.[3]

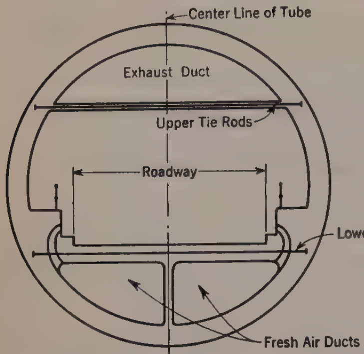

FIG. 1.—TYPICAL SECTION OF SUBAQUEOUS TUNNEL TUBE.

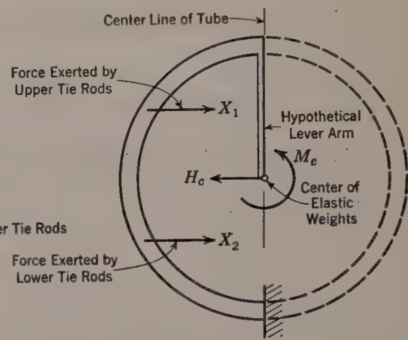

FIG. 2.—DIAGRAM OF BASIC TUBE.

The equations developed in this paper refer to the analysis of stress in a reinforced concrete tube. However, they may also be used for the analysis of stresses in a cast-iron tube by substituting the proper values for the modulus of elasticity. The basic assumptions made for the elastic materials generally used in the theory of elasticity were adopted for this paper. However, the possible effect of plastic flow in concrete was not considered.

Notation.—The letter symbols in this paper are introduced in the text as they occur and are summarized for reference in the Appendix. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials"[4] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

DERIVATIONS

The loading sustained by a tunnel tube consists of external earth and water loads, the weight of the structure, the live loads on the roadway deck, and the force of buoyancy. The ring and the loads acting on it are generally symmetrical about a vertical axis through the crown section, and, consequently, the stresses involved are also symmetrical about the vertical axis. The shearing stresses at the upper and lower crown sections are equal to zero. In developing the equations the half-ring shown in Fig. 2 is analyzed, representing a 1-ft length of tube, fixed at the bottom, with the right half removed. The tie-rods are replaced by tensile forces, and the reactions from the right half are expressed by the horizontal force and bending moment applied at the elastic center of the tube.

An elastic tube with two horizontal flexible tie-rods is statically indeterminate in the fourth degree; that is, there is a deficiency of four equations for solution as a statically determinate problem. The following are the con-

[3] *Engineering News-Record,* December 3, 1925, p. 902.
[4] A.S.A.—Z10*a*—1932.

ditions to be met: (1) From symmetry, the vertical diameter in Fig. 2 may be assumed as fixed; (2) the horizontal displacement and the rotation of the crown sections may be equated to zero; and (3) the horizontal displacements of the ring, at the points where the tie-rods are connected are equal to the strains in the corresponding ties.

Case 1.—Tunnel Section Without Tie-Rods.—Equations developed for determining stresses in an elastic rib arch fixed at the ends,[5] may be adapted to the solution of stresses in any closed ring without tie-rods. Since such a ring is symmetrical about the vertical axis, its elastic center will be on the vertical axis and its distance from the upper crown section, y_c, may be computed from the equation,

$$y_c = \frac{\int h_y \, dw}{\int dw} \quad \text{.............(1)}$$

in which (see Fig. 3), h_y = the vertical distance from the axis at the crown, to any point, x, y, on the ring; $dw = \frac{ds}{E_c I}$, the elastic weight of the arch element; ds = the length of an element measured along the neutral axis; E_c = the modulus of elasticity of the concrete in compression; and I = the moment of inertia of the section of the tube.

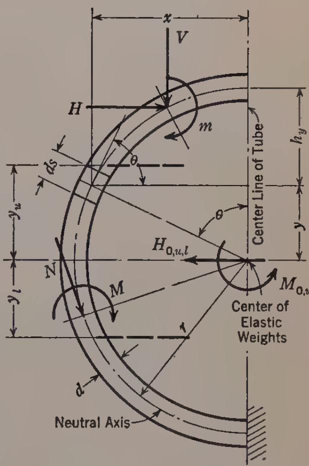

Fig. 3.—Nomenclature.

The horizontal force, H_0, that expresses the effect of the right-half ring (transmitted through the hypothetical arm shown in Fig. 2) is determined from:

$$H_0 = \frac{\int M \, y \, dw + \int H \cos^2\theta \, dv + \int V \sin\theta \cos\theta \, dv}{\int y^2 \, dw + \int \cos^2\theta \, dv} \quad \text{........(2)}$$

in which, M = the bending moment at some point, x, y, on the neutral axis of the tube produced by the forces applied between that point and the crown; H and V are the horizontal and vertical components of the sum of all the forces between Point x, y, and the crown; $dv = \frac{ds}{E_c A}$; A = the area of the section cut by a radial plane, per linear foot of tunnel; and θ = the angular distance of Point x, y, measured counter-clockwise. The moment, M_0, at the elastic center, introduced to allow for the removal of one-half the tunnel tube, is:

$$M_0 = \frac{\int M \, dw}{\int dw} \quad \text{..........................(3)}$$

[5] "Movable and Long-Span Steel Bridges", by G. A. Hool and W. S. Kinne, p. 440.

The horizontal distortion at Point x, y, (see Fig. 4) may be determined by:

$$\Delta_{0u} = \int M\, y\, dw + \int H \cos^2\theta\, dv + \int V \sin\theta \cos\theta\, dv$$
$$- H_0 \left(\int y^2\, dw + \int \cos^2\theta\, dv \right) - M_0 \int y\, dw \quad \ldots\ldots\ldots\ldots (4)$$

the integration being taken from the fixed support to Point x, y. When the external load is a unit horizontal force applied at the upper tie-rod joint (Fig. 4(*b*)), the horizontal and vertical components are $H = 1$, and $V = 0$,

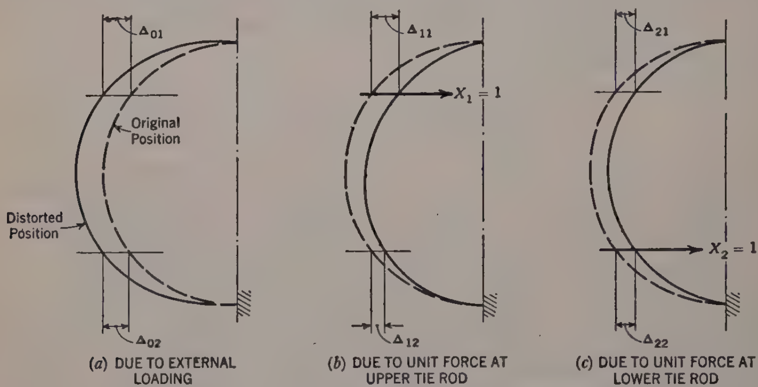

FIG. 4.—DIAGRAMS OF HORIZONTAL DISTORTION.

and the bending moment at Point x, y (Fig. 3), on the neutral axis below the upper tie-rod joint is,

$$M = y_u - y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, y_u = the vertical distance of the upper tie-rod joint from the elastic center. Therefore, from Equation (2), the horizontal reaction at the elastic center, produced by a unit force at the upper tie-rod joint is:

$$H_u = \frac{\int y\,(y_u - y)\, dw + \int \cos^2\theta\, dv}{\int y^2\, dw + \int \cos^2\theta\, dv} \quad \ldots\ldots\ldots\ldots (6)$$

and the reaction bending moment, from Equation (3) is,

$$M_u = \frac{\int (y_u - y)\, dw}{\int dw} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

From Equation (5) the horizontal distortion at some point, x, y, on the neutral axis, produced by a unit force acting along the upper tie-rod is:

$$\Delta_{1u} = \int y\,(y_u - y)\, dw + \int \cos^2\theta\, dv$$
$$- H_u \left(\int y^2\, dw + \int \cos^2\theta\, dv \right) - M_u \int y\, dw \quad \ldots\ldots\ldots (8)$$

Likewise, the horizontal thrust, H_l, and the bending moment, M_l, at the elastic center of the ring produced by a unit force applied along the lower tie-rod, are:

$$H_l = \frac{\int y\,(y_l - y)\,dw + \int \cos^2\theta\,dv}{\int y^2\,dw + \int \cos^2\theta\,dv} \quad \dots\dots\dots\dots\dots\dots (9)$$

and,

$$M_l = \frac{\int (y_l - y)\,dw}{\int dw} \quad \dots\dots\dots\dots\dots\dots (10)$$

in which, y_l = the vertical distance from the elastic center to the lower tie-rod. The horizontal distortion, Δ_{1l}, of some point, x, y, produced by a unit force applied along the lower tie-rod is the same as Equation (8), substituting for y_u, H_u, and M_u, the corresponding values of y_l, H_l, and M_l.

The limit of integration for the nominators in Equations (6), (7), (9), and (10), is from the intersection of the axis at the invert crown to the point, x, y, at which the tie-rod is fastened. The limits of integration for Equation (8) is from the intersection of the axis at the invert to the point, x, y, at which the distortion is computed.

Case 2.—Tunnel Section with Horizontal Tie-Rods.—Whereas, a circular, elastic tube with two horizontal tie-rods is statically indeterminate in the fourth degree, without the tie-rods the section is statically indeterminate in the second degree, and in the latter case, the reaction forces may be computed by Equations (2) and (3). Stresses in the tie-rods may be determined from the Maxwell theorem of reciprocal displacements.

The horizontal distortions at the level of the upper and lower tie-rods are, respectively, Δ_{ou} and Δ_{ol}. In the tube with ties these distortions are corrected by tensile forces in the tie-rods. The final force in the upper tie-rod, assisting the tube to carry the external loads, is indicated by X_u and at the lower rod by X_l. The horizontal distortion at the upper tie-rod joint produced by the upper tie-rod stress is $X_u \Delta_{1u}$, and the distortion produced by the lower tie-rod stress is $X_l\,\Delta_{1l}$. The total distortion of the ring at the level of the upper tie-rod, from its original position, is equal to one-half the strain in that tie-rod, or

$$\Delta_1 = \frac{X_u\,L_u}{2\,E_s\,A_u} \quad \dots\dots\dots\dots\dots\dots (11)$$

(assuming no initial tension in the tie-rod). Therefore, from the Maxwell theorem the equation may be written:

$$\frac{X_u\,L_u}{2\,E_s\,A_u} = X_u\,\Delta_{1u} + X_l\,\Delta_{1l} + \Delta_{ou} \quad \dots\dots\dots\dots\dots\dots (12)$$

Likewise, for the lower tie-rod an equation of distortion may be written:

$$\frac{X_l\,L_l}{2\,E_s\,A_l} = X_l\,\Delta_{2l} + X_u\,\Delta_{2u} + \Delta_{ol} \quad \dots\dots\dots\dots\dots\dots (13)$$

in which, A_u and A_l are the sectional areas of the upper and lower tie-rods, respectively. From Maxwell's theorem $\Delta_{1l} = \Delta_{2u}$. Solving, simultaneously, Equations (12) and (13) the reactions, X_u and X_l, in the tie-rods may be determined.

The horizontal thrust, H_c, and the bending moment, M_c, at the elastic center of the tube, with tie-rods, may be computed from:

$$H_c = H_0 + X_u H_u + X_l H_l \qquad (14)$$

and,

$$M_c = M_0 + X_u M_u + X_l M_l \qquad (15)$$

The bending moment, M, and the normal thrust, N, at any radial section may be computed from statics; thus, for points on the axis of the ring between the upper tie-rod and the horizontal diameter of tube:

$$M_q = - M_c + H_c y + M + X_u (y_u - y) \qquad (16)$$

and,

$$N = H_0 \cos\theta - H \cos\theta + V \sin\theta - X_u \cos\theta \qquad (17)$$

Case 3.—Temperature Stresses.—Assuming the tie-rod removed, the horizontal thrust at the elastic center, produced by a change (t) in the temperature of the atmosphere is expressed by:

$$H_t = \frac{c\, t\, r\, E_c}{\int y^2\, dw + \int \cos^2\theta\, dv} \qquad (18)$$

in which, c = a coefficient of temperature expansion or contraction of concrete for 1° variation in temperature; and r = the radius of the neutral axis of the tube.

The horizontal distortion of some point, x, y, on the neutral axis of the tube for a change of temperature of $t°$ is expressed by:

$$\Delta_t = c\, t\, r\, E_c - H_t \left(\int y^2\, dw + \int \cos^2\theta\, dv \right) \qquad (19)$$

Tension or slack in the upper and lower tie-rods produced by temperature changes in the atmosphere may be computed from Equations (12) and (13), except that Δ_{01} and Δ_{02} must be changed to the temperature distortions, Δ_{tu} and Δ_{tl}, in the upper and lower rods. Temperature stresses may be computed also for differences in temperature between rods and ring, with appropriate distortions, Δ_{tu} and Δ_{tl}. It should be noted that the earth pressure loads may also change with changes in temperature.

Case 4.—Circular Tunnel Tube of Uniform Thickness.—The equations for computing stresses in a circular ring of uniform thickness may be simplified considerably. The elastic center of the tube will be in the geometric

center of the ring. The co-ordinates of a radial section may be expressed by angular functions, thus,

$$x = r \sin \theta \quad \text{.......................(20)}$$

$$y = r \cos \theta \quad \text{.......................(21)}$$

$$ds = r \, d\theta \quad \text{.......................(22)}$$

$$dw = K_1 \, r \, d\theta \quad \text{.......................(23)}$$

and,

$$dv = K_2 \, r \, d\theta \quad \text{.......................(24)}$$

in which, $K_1 = \dfrac{1}{E_c \, I}$; and, $K_2 = \dfrac{1}{E_c A}$. Substituting Equations (20) to (24) in Equations (2), (3), and (4):

$$H_0 = \frac{r \, K_1 \int M \cos \theta \, d\theta + K_2 \int H \cos^2 \theta \, d\theta + K_2 \int V \sin \theta \cos \theta \, d\theta}{(r^2 K_1 + K_2) \int \cos^2 \theta \, d\theta} \quad \text{....(25)}$$

$$M_0 = \frac{\int M \, d\theta}{\int d\theta} \quad \text{.......................(26)}$$

and,

$$\Delta_0 = r \, K_1 \int M \cos \theta \, d\theta + K_2 \int H \cos^2 \theta \, d\theta + K_2 \int V \sin \theta \cos \theta \, d\theta$$

$$- (K_1 r^2 + K_2) \int \cos^2 \theta \, d\theta - r \, K_1 \, M_0 \int \cos \theta \, d\theta \quad \text{.........(27)}$$

Example 1

As an illustration of the practical application of the equations developed, the stresses in the elastic tunnel tube with the tie-rods in the ceiling were computed. Fig. 5 shows the dimensions of the tube and the external loads acting on it. In addition, the sectional area of the tie-rods is $A_u = 1.3$ sq in. per lin ft of tunnel the length of the tie-rods is $L_u = 29.67$ ft; and the vertical distance to the tie-rod from the center of the tube is $y_u = 8.80$ ft.

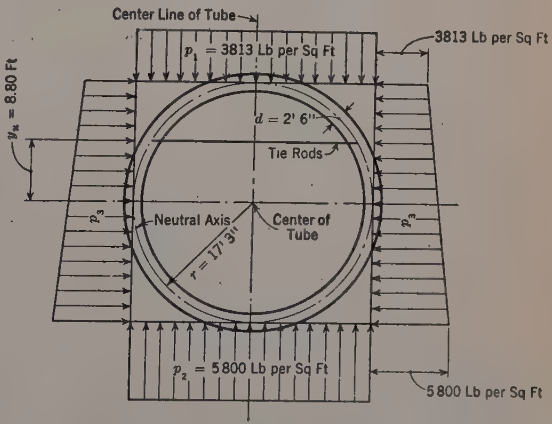

FIG. 5.—TUBE SECTION AND LOADING USED IN EXAMPLE.

From symmetry it follows that the elastic center of the tube corresponds to the geometric center of the ring. The horizontal thrust, H_o, and the bending moment, M_o, produced by the external loading, as computed by Equations (25) and (26), are $H_o = 81\,500$ lb and $M_o = 1\,450\,000$ ft-lb. The horizontal distortion, Δ_{o1}, at the point, y_u, (with the tie-rods removed), due to the external loading, is computed by Equation (27): $\Delta_{ou} = 0.248$ ft. The horizontal thrust, H_u, and bending moment, M_u, at the elastic center produced by a horizontal load of 1 000 lb at the point, y_u, as computed by Equations (6) and (7) are: $H_u = 668$ lb, and $M_u = 10\,600$ ft-lb. The horizontal distortion of the tie-rod joint of the tube produced by a horizontal force of 1 000 lb at y_u, as computed by Equation (8) is, $\Delta_{1u} = 0.011$ ft. The strain in the tie-rod produced by a tensile force of 1 000 lb, is: $\Delta_u = \dfrac{1\,000 \times L_u}{2\,E_s\,A_u} = 0.00038$ ft.

Substituting the values of the distortions in Equation (12) and solving, the unknown reaction in the tie-rods is found to be $X_u = 22\,877$ lb. In Equation (12), for the present example, $X_l = 0$. The unit stress in the tie-rods is: $f_s = \dfrac{X_u}{A_u} = \dfrac{22\,877}{1.3} = 17\,598$ lb per sq in. The effect of the direct stresses on the reactions at the elastic center and the horizontal distortions of the tube have not been considered in this example.

Summary

The advantages of the equations based on the Maxwell theorem in computing the stresses in a tunnel tube may be summarized, as follows:

(1) The stresses in the shell and tie-rods of a tunnel may be computed by an adaptation of Maxwell's theorem with the same precision as by any other refined method based on the theorem of least work.

(2) The significance of the various terms in the equations for computing the stresses may easily be visualized. Therefore, the computation of stresses may be simplified by reducing them for negligibly small terms. Furthermore, the simplicity of interpreting the signs of the expressions for distortion at tie-rods is self-evident.

(3) The "trial-and-error" changes in the dimensions of a tunnel tube and the tie-rods in the method based on Maxwell's theorem will affect a smaller number of the terms than is the case with the method of least work.

(4) Further simplifications may be introduced by preparing tables and diagrams for the geometrical constants involved.

(5) The method of computing stresses in the tube based on Maxwell's theorem has a wide application. It may readily be used in computing stresses in any other type of statically indeterminate structures.

The necessity for a shorter and more accurate method of computing stresses in a tunnel tube is unquestionable. In this respect, the method presented herein, based on Maxwell's theorem, will be of interest to the designers of tunnels.

APPENDIX

Notation

The following symbols, defined where first introduced in the paper, are re-arranged herein for convenience of reference:

A = area of a radial section of a 1-ft length of tunnel; also, with subscripts, A = cross-sectional area of a tie-rod per linear foot of tunnel, A_u referring to the upper of two rods, and A_l to the lower rod;

c = coefficient of expansion or contraction of concrete for a change in atmosphere of 1° F;

d = thickness of the tunnel ring;

E = modulus of elasticity: E_c, for concrete in compression; and E_s, for steel in tension;

f = unit stress, f_s, referring to unit steel stress;

H = horizontal thrust or reaction; horizontal component of force at any point, x, y, produced by forces on the ring between that point and the crown; H_c = horizontal component of force at the crown section, transferred to the center of the elastic weights; H_o = horizontal component of force applied at the center of the elastic weights to allow for the removal of one-half the tunnel section; H_u = a horizontal force at the center of the elastic weights produced by a unit force applied along the upper tie-rod; H_l = a horizontal force at the center of the elastic weights, applied along the lower tie-rod; H_t = a horizontal thrust at the center of the elastic weights, produced by a change, t, in the temperature of the atmosphere;

h = height; h_y = the vertical distance from the crown to any point, x, y, on the axis of the tunnel ring;

I = moment of inertia of a radial section cut from a 1-ft length of tunnel;

K = a constant; $K_1 = \dfrac{1}{E_c\, I}$; and $K_2 = \dfrac{1}{E_c A}$;

L = length of a tie-rod between its intersection with the axis of the tunnel ring, L_u referring to the upper of two rods and L_l to the lower rod;

M = moment; the bending moment at any point, x, y, produced by the forces on the ring between that point and the crown; M_c = moment at the crown, transferred to the center of the elastic weights; M_o = moment applied at the center of the elastic weights to compensate for the removal of one-half the ring; M_u and M_l = moment at the center of the elastic weights produced by a unit rotation in the tunnel ring at y_u and y_l, respectively; and M_t = moment at the center of the elastic weights produced by a change, t, in the temperature of the atmosphere; M_q = the bending moment at any point of the tunnel tube with tie-rods, Equation (16).

N = component of thrust normal to a radial plane through any point of the tunnel tube;

p = pressure per unit area; on the 1-ft length of tunnel, p_1, p_2, and p_3, = the uniformly distributed loads on the top, bottom, and sides, respectively, of the tunnel ring (see Fig. 5);

r = radius of a circle described from the center of elastic weights; the radius of the geometric center of the neutral axis of a circular tunnel section;

s = linear distance, measured along the axis of a tunnel section; ds = an element of s;

t = change in the temperature of the atmosphere, in degrees Fahrenheit;

V = vertical component of force at any point, x, y, due to forces on the ring between that point and the crown;

dv = elastic weight with normal stress $= \dfrac{ds}{E_c A}$;

w = weight; dw = elastic weight $= \dfrac{ds}{E_c I}$;

X = a tie-rod force, or reaction, X_u referring to the upper tie-rod and X_l to the lower one;

x = a horizontal distance measured from the center of elastic weights;

y = a vertical distance measured from the center of elastic weights; y_c = the distance to the neutral axis of the tunnel ring at the crown; y_u = the distance to the upper tie-rod; y_l = the distance to the lower tie-rod;

Δ = horizontal distortion or displacement; Δ_0 = the distortion at any point x, y, due to external loading; Δ_u = the distortion in the tunnel ring, at Point y_u; Δ_1 = the distortion at any point, x, y, due to a unit load applied along the upper tie-rod; Δ_2 = the distortion at any point, x, y, due to a unit load applied along the lower tie-rod; Δ_l = the distortion in the tunnel ring at y_l; Δ_t = distortion at any point, x, y, due to a change in the temperature of the atmosphere; Δ_{01} = the distortion in the tunnel ring at Point y_u due to strain in the upper tie-rod $= \dfrac{X_u L_u}{2 E_s A_u}$; Δ_{02} = the distortion in the tunnel ring at Point y_l due to strain in the lower tie-rod $= \dfrac{X_l L_l}{2 E_s A_l}$;

θ = angular distance; the angle between a horizontal line through any point, x, y, and the tangent to the neutral axis at that point; in a circular tunnel section the angular distance (central angle) to Point x, y, measured counter-clockwise.

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PAPERS

DEFLECTIONS BY GEOMETRY

By David B. Hall[1], Assoc. M. Am. Soc. C. E.

Synopsis

The fundamental method for finding deflections is by geometry, rather than by the law of conservation of energy or the principle of least work. The few simple rules that are necessary in a geometrical analysis are given in this paper, and are illustrated by developing methods for calculating influence lines for such structures as arches, and deflections in trusses.

General Theory

For the engineer who is constantly dealing with stresses, but meets only occasionally with deflections or statically indeterminate structures, the methods of calculating deflections in terms of imaginary forces are of great assistance. They enable him to make computations with a minimum knowledge of the subject of deflections. For this same reason, however, such indirect methods as that of least work, or of elastic weights, tend to stand in the way of acquiring real facility in the subject.

A simple illustration will show the real inward nature of the problem. Consider the four following problems.

Problem 1.—Determine the Force X, in Fig. 1, by the Principles of Statics.—The solution is evidently to take moments about the support; whence $X = 2$ lb.

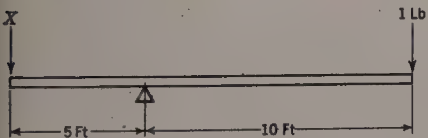

Fig. 1.—Mechanics Problem.

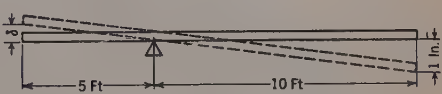

Fig. 2.—Geometry Problem.

Problem 2.—Determine the Distance, δ_x, in Fig. 2, by Geometry.—From similar triangles it is evident that $\delta = 0.5$ in.

Problem 3.—Determine the Force, X, Fig. 1, in Some Manner Other Than by Statics.—If the force of 1 lb is balanced by the unknown force, X, and the lever is rotated slightly, one side will gain, and the other side will lose, energy. According to the law of conservation of energy, these two changes must

Note.—Discussion on this paper will be closed in April, 1937, *Proceedings.*

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be equal. Therefore, continuing from the solution of Problem 2, $X \times 0.5$ in. $= 1$ lb $\times 1$ in.; and, $X = 2$ lb.

Problem 4.—Determine the Deflection δ, Fig. 2, in Some Manner Other Than by Geometry.—Once more using the law of conservation of energy, apply a force on the left end and find by statics the force on the right end to balance it; then find how far the first force must move to balance the energy gained or lost by the second force in moving 1 in. This distance will be 0.5 in.

Comment.—In the foregoing group of problems it is quite obvious that the simplest solution of the mechanics problem is by mechanics and the simplest solution of the geometrical problem is by geometry. The solution presented in Problem 4, however, is often referred to as the fundamental method of solving deflection problems. It is none other than the method of work. By the more usual method of presentation, the force corresponding to that on the right side of the lever would be furnished by elastic members, and, therefore, both it and the force on the left would be functions of the deflections and the energy would be determined by integral calculus, although these complications seem to be irrelevant.

The group of problems suggests one constructive idea, however: When new methods for applying the geometry of deflections are being sought, there will usually be similar familiar methods for solving corresponding mechanics problems to serve, not as substitutes, but as guides.

Probably the most important theorem of the geometry of deflections is one which bears a very close resemblance to the principle of moments:

Theorem 1.—If a body rotates about a point through an infinitesimal angle (in ordinary structures all deflections are treated as infinitesimal quantities), the component in any direction of the movement of any point in the body is proportional to the perpendicular distance from the line of that component to the center of rotation.

The proof of Theorem 1 is quite simple. In Fig. 3 it will be seen, in the first place, that the actual motion of a point such as A is at right angles to the radius, OA, and proportional to its length. (The motion is numerically equal to $\overline{OA} \times \alpha$.) If AC is the desired component of this movement, and OD is the radius drawn perpendicular to it, the triangles, OAD and ABC, are similar, and $\frac{\overline{AC}}{\overline{AB}} = \frac{\overline{OD}}{\overline{OA}}$; or,

$$\overline{AC} = \frac{\overline{OA} \times \alpha \times \overline{OD}}{\overline{OA}} = OD \times \alpha.$$

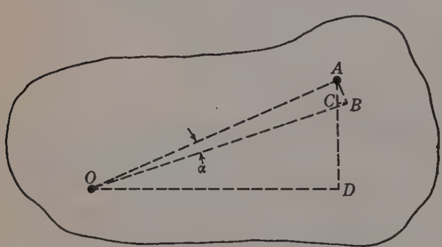

FIG. 3.—ROTATION THEOREM

When dealing with deflections in terms of vertical and horizontal components, the foregoing theorem can be stated more conveniently as follows:

Theorem 2.—When a body rotates about a point, the vertical displacement of any point is proportional to its horizontal distance from the center of rotation, and the horizontal displacement is proportional to its vertical distance.

Deflections of Beams

The simplest type of beam to investigate is a cantilever. Consider the beam in Fig. 4. Suppose that the moment in the shaded area causes this portion of the beam to bend through an angle, α. (This angle can be calculated by the well known formula, $\alpha = \frac{M\ dL}{E\ I}$, in which dL is the length of the portion considered; M is the moment; E, the modulus of elasticity; and I, the moment of inertia.) Then Theorem 2 can be applied directly to determine the deflection at the end contributed by this part of the beam. The vertical deflection is $x\,\alpha$, or $\frac{M\ dL}{E\ I}\,x$, and the horizontal deflection is $\frac{M\ dL}{E\ I}\,y$. To find the complete deflection, it is necessary to add the contributions from each part of the beam, integrating by calculus if possible, or by dividing the beam into a number of sections and treating the average properties of each section.

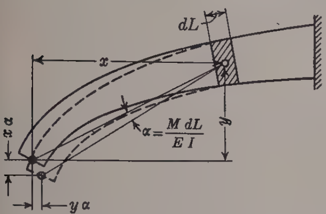

Fig. 4.—Deflection of a Cantilever.

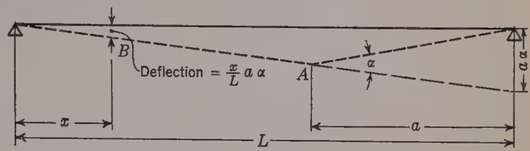

Fig. 5.—Defldction of a Simple Beam.

To find the moments in a cantilever it is only necessary to multiply the loads and their lever arms. To find the moments in a simple beam an additional step is required: (1) Take moments about one support and find one reaction; and (2) find the moment caused by this reaction. Similarly, an extra step is required in finding deflections in a simple beam. The process is illustrated in Fig. 5. Bending through an angle, $\alpha \left(=\frac{M\ dL}{E\,I}\right)$, at Point A would produce an upward deflection, $a\,\alpha$, at the right end if the left part were held fast. If the beam were then rotated as a whole until the right end rested on its support, the beam would assume the position which it actually does occupy, and the proportionate deflection at Point B would be $\frac{x}{L}\,a\,\alpha$.

In both of the foregoing examples it will be discovered that the coefficient of $\frac{M\ dL}{E\ I}$ is identical to the moment, m, which a unit load applied where the deflection is measured would produce at the point that is bending. This again illustrates the connection between the methods of Problems 2 and 4, and forms the basis of the dummy load formula:

$$\delta = \int \frac{M\ m\ dL}{E\ I} \qquad \qquad (1)$$

There are two general methods of determining deflections: The foregoing, which might be referred to as single integration, and the method of double integration. The second method is particularly adapted to determining deflections at all the division points of an irregular beam such as an arch. In order to have something definite and clear cut upon which to perform the various operations that are involved, the beam should be conceived of in the following manner:

1.—Cut the beam into divisions as indicated by the dotted lines in Fig. 6. (The dotted lines are necessary for the explanation but are not essential in actual design.)

2.—Set a point in the middle of each division. When the beam is subjected to bending the faces of Division 4, say, are turned through a certain angle relative to each other, the part of the axis in this division being bent in a curve. This angle is $\frac{M\ dL}{E\ I}$, in which dL is the length of the division. The same happens in each of the other divisions.

3.—Instead of considering the divisions curved by the moments, the beam is assumed to consist of a series of straight lines from point to point and the bending to consist of a change in the angles at the joints in these lines. With the beam thus articulated it is a simple matter to visualize any operation that has to be performed.

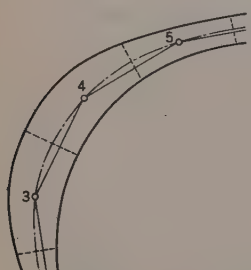

FIG. 6.

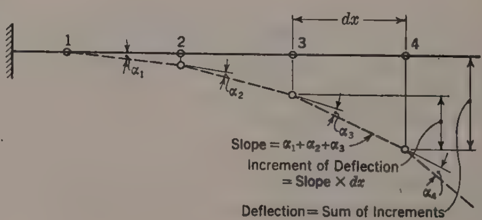

FIG. 7.—DEFLECTION BY DOUBLE INTEGRATION.

The deflection of a cantilever by double integration is illustrated in Fig. 7. The process is exactly the same for a curved beam. A straight beam was illustrated to avoid confusing the reader with two sets of angles. For any other beam than a cantilever the best method is to determine the rotation at some point first and then proceed from there to sum up the angles at the joints. For a symmetrical simple beam, symmetrically loaded, this is accomplished by starting at the center where the slope is zero. It may be noted that the point of initial slope and the point of zero deflection do not need to be the same; the first summation can proceed in one direction, and the second in the opposite direction.

Influence Lines for Arches.—Calculation of the influence line for the thrust in a symmetrical two-hinged arch (see Fig. 8) is a good illustration of the foregoing method. Instead of placing a unit load at successive points

to find the displacement of the hinges, the vertical deflections of points along the arch due to a unit thrust at the hinges may be calculated, since by the theorem of reciprocal displacements both are alike. In Table 1 it will be

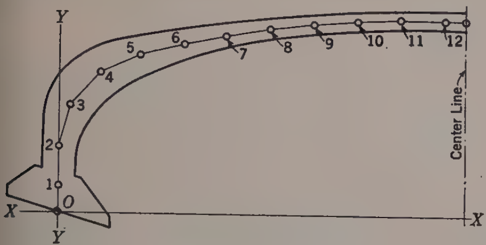

FIG. 8.—TWO-HINGED ARCH.

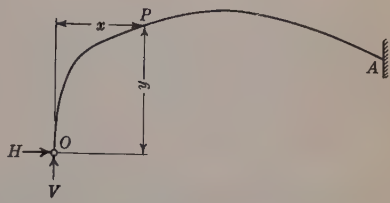

FIG. 9.—MATHEMATICAL EQUIVALENT OF THE BEGGS METHOD.

noted that the lines are double-spaced. Entries opposite the points refer in some manner to the individual points (co-ordinates, angles, etc.), whereas entries between points refer to the lines joining them (slopes, increments

TABLE 1.—INFLUENCE LINE FOR TWO-HINGED ARCH (SEE FIG. 8)

Point	Abscissa, x (1)	Element, dx (2)	Thickness of arch, t (3)	t^3 (4)	Ordinate, y (5)	Horizontal deflection, $\frac{y^2}{t^3}$ (6)	Angle, $\frac{y}{t^3}$ (7)	Slope, $\Sigma \frac{y}{t^3}$ (from crown) (8)	Deflection increment, Column (8) $\times$ Column (2) (9)	Vertical deflection, Σ Column (9) (from hinge) (10)	H influence factors, $\frac{\text{Column (10)}}{627.2}$ (11)
1......	0		4±	100±	2.2	0.0	0.02			0	0
		0.0						16.32	0.0		
2......	0		3.70	50.7	6.6	0.9	0.13			0	0
		1.1						16.19	17.8		
3......	1.1		4.90	117.6	10.9	1.0	0.09			17.8	0.028
		3.0						16.10	48.3		
4......	4.1		5.12	134.2	14.1	1.5	0.10			66.1	0.105
		4.1						16.00	65.5		
5......	8.2		3.96	62.1	15.9	4.1	0.26			131.6	0.210
		4.3						15.74	67.7		
6......	12.5		3.00	27.0	17.1	10.8	0.63			199.3	0.318
		4.3						15.11	65.0		
7......	16.8		2.46	14.9	18.0	21.8	1.21			264.3	0.421
		4.4						13.90	61.1		
8......	21.2		2.13	9.66	18.75	36.4	1.94			325.4	0.519
		4.5						11.96	53.8		
9......	25.7		1.96	7.53	19.3	49.5	2.56			379.2	0.604
		4.4						9.40	41.4		
10......	30.1		1.88	6.64	19.7	58.5	2.96			420.6	0.670
		4.4						6.44	28.4		
11......	34.5		1.85	6.33	20.0	63.2	3.16			449.0	0.715
		4.5						3.28	14.8		
12......	39.0		1.83	6.13	20.1	65.9	3.28			463.8	0.739
		2.2						0	0		
Crown...	41.2		....	...	20.1	...	..			463.8	0.739
$\Sigma \frac{y^2}{t^3}$	....	...	....	...	...	313.6	..	....	......	....	....
$2 = \Sigma \frac{y^2}{t^3}$	....	...	....	...	...	627.2	..	.. ...	......	....	....

of deflection, etc.). For divisions of constant length, $\frac{dL}{EI}$ is proportional to $\frac{1}{t^3}$, in which t is the thickness of the section. For a unit thrust, $M = y$, and

$\frac{M\ dL}{E\ I}$ is equivalent to $\frac{y}{t^3}$. The horizontal displacement between the hinges is equal to $\sum \frac{y^2}{t^3}$, and the influence line is obtained by dividing the deflections for unit loads by this displacement.

As these operations become more familiar, increasingly difficult problems can be solved. For example, apply a horizontal force of 1 lb at Point O, in Fig. 9. The moment at any point, P, will be 1 lb $\times$ y, and the movement at Point O will be, $\sum \frac{y^2\, dL}{E\, I}$, horizontally, and $\sum \frac{x\, y\, dL}{E\, I}$, vertically. Similarly, a vertical force of 1 lb will produce a vertical movement of $\sum \frac{x^2\, dL}{E\, I}$ and a horizontal movement of $\sum \frac{x\, y\, dL}{E\, I}$.

When these deflections have been determined it is a simple matter to compute a combination of vertical and horizontal forces at Point O which will cause a horizontal deflection of one unit and no vertical deflection. The formulas for this condition, and for a unit vertical deflection, are as follows:

For unit horizontal deflections:

$$H = \frac{\Sigma\ (x^2\ G)}{B} \qquad \text{(2)}$$

and,

$$V = \frac{\Sigma\ (x\ y\ G)}{B} \qquad \text{(3)}$$

and, for vertical deflections:

$$H = \frac{\Sigma\ (x\ y\ G)}{B} \qquad \text{(4)}$$

and,

$$V = \frac{\Sigma\ (y^2\ G)}{B} \qquad \text{(5)}$$

in which $G = \frac{dL}{E\ I}$; and.

$$B = \Sigma\ (x^2\ G)\ \Sigma\ (y^2\ G) - \left[\Sigma\ (x\ y\ G)\right]^2 \qquad \text{(6)}$$

Furthermore, it is not difficult to compute the moments and, in turn, the deflections at a succession of points along the structure due to these forces at Point O. These deflections constitute the influence line for the horizontal reaction at Point O in a structure fixed at Point A (Fig. 9) and hinged at Point O, because the foregoing process is the mathematical equivalent of the Beggs mechanical method for obtaining influence lines.

This principle will be illustrated by the calculation of the constants for unsymmetrical beams of variable section. If the right end of Beam AB

(Fig. 10(*a*)) is fixed and the left end is rotated through a unit angle, the resulting end moments can be called stiffness factors. The moment at End A will be denoted as K_A; the moment at End B will be denoted as K_{AB} (or K_{BA},

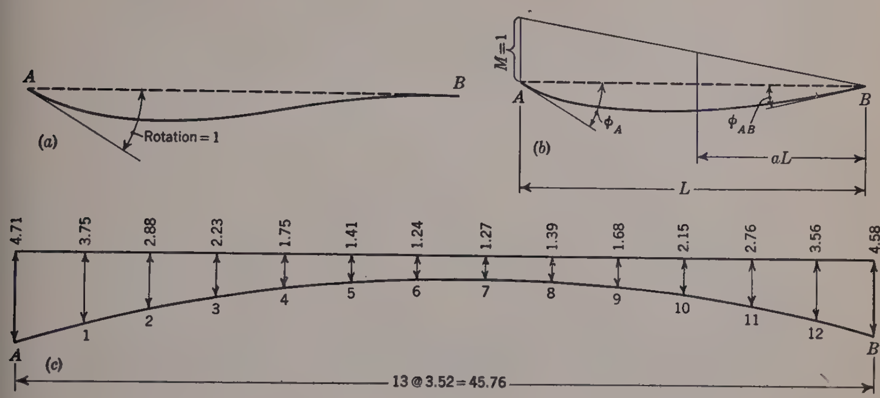

FIG. 10.

since it will be found that the fixed-end moment for either end, when the opposite end is rotated, is the same). Similarly, if End A is fixed and End B is given a unit rotation, the moment at End B will be K_B.

For the moment distribution method, K_A and K_B are the usual stiffness factors, and $\frac{K_{AB}}{K_A}$ and $\frac{K_{AB}}{K_B}$ are the carry-over factors. For the slope deflection method,

$$M_{AB} = K_A (\theta_A - R) + K_{AB} (\theta_B - R) + C_{AB} \ldots\ldots\ldots\ldots (7)$$

and,

$$M_{BA} = K_B (\theta_B - R) + K_{AB} (\theta_A - R) + C_{BA} \ldots\ldots\ldots\ldots (8)$$

in which C_{AB} and C_{BA} are fixed beam moments, the signs being algebraic.

The method of obtaining the foregoing K-factors is as follows: The clockwise rotation of End A in the simple beam in Fig. 10(*b*) is equal to the deflection of End B from the tangent at End A, divided by L, and can readily be shown to be:

$$\phi_A = \frac{\int a^2\, dL}{E\, I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

or for finite divisions,

$$\phi_A = \frac{\Sigma\, a^2\, dL}{E\, I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

This rotation will be referred to as ϕ_A. The counterclockwise rotation at Point B, is equal to:

$$\theta_{AB} = \frac{\Sigma\, a\,(1 - a)\, dL}{E\, I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

For a unit moment at Point B, $\phi_B = \dfrac{\Sigma\,(1-a)^2\,dL}{E\,I}$, and ϕ_{BA} is the same as ϕ_{AB}. To obtain the stiffness factors, K_A and K_{AB}, it is simply necessary to make the rotation at Point A equal to unity and the rotation at Point B equal to zero, thus producing the condition illustrated in Fig. 10(a), which defined these factors. The necessary equations are:

$$M_A\,\phi_A - M_{AB}\,\phi_{AB} = 1 \quad\ldots\ldots(12)$$

and,

$$M_A\,\phi_{AB} - M_{AB}\,\phi_B = 0 \quad\ldots\ldots(13)$$

and the solution gives,

$$M_A = K_A = \frac{\phi_B}{\phi_A\,\phi_B - (\phi_{AB})^2} = \frac{\phi_B}{D} \quad\ldots\ldots(14)$$

and,

$$M_{AB} = K_{AB} = \frac{\phi_A\,B}{D} \quad\ldots\ldots(15)$$

Similarly,

$$K_B = \frac{\phi_A}{D} \quad\ldots\ldots(16)$$

A typical beam is shown in Fig. 10(c), and the evaluation of these constants for this beam is contained in Table 2. It will be noticed that multiplying or

TABLE 2.—EVALUATION OF CONSTANTS FOR AN UNSYMMETRICAL BEAM (SEE FIG. 10(c))

Point	Elastic factor, $\frac{dL}{t^3}$ (1)	Values of $(13a)^2$ (2)	Values of $13a$ $(13-13a)$ (3)	Values of $(13-13a)^2$ (4)	Column (1) × Column (2) $= 13^2\,\phi_A$ (5)	Column (1) × Column (3) $= 13^2\,\phi_{AB}$ (6)	Column (1) × Column (4) $= 13^2\,\phi_B$ (7)
A*	0.017	169	0	0	2.87	0.00	0.00
1	0.067	144	12	1	9.65	0.80	0.07
2	0.147	121	22	4	17.79	3.23	0.59
3	0.317	100	30	9	31.70	9.51	2.85
4	0.657	81	36	16	53.22	23.65	10.51
5	1.256	64	40	25	80.38	50.24	31.40
6	1.846	49	42	36	90.45	77.53	66.46
7	1.718	36	42	49	61.85	72.16	84.18
8	1.311	25	40	64	32.78	52.44	83.90
9	0.742	16	36	81	11.87	26.71	60.10
10	0.354	9	30	100	3.19	10.62	35.40
11	0.167	4	22	121	0.67	3.67	20.21
12	0.078	1	12	144	0.08	0.94	11.23
B*	0.018	0	0	169	0.00	0.00	3.04
Summation...	.....	...	..	...	396.50	331.50	409.94

* One-half section.

dividing at strategic points by the number of divisions (13) facilitates the numerical work. Referring to the summation of Table 2:

$$13^4\,\phi_A\,\phi_B = \text{product of Columns (5) and (7)} = 16\,254.1$$
$$13^4\,\phi_{AB}^2 = \text{Column (6) squared} = 10\,989.2$$

$$13^4\,D = 13^4\,(\phi_A\,\phi_B - \phi_{AB}^2) = 5\,264.9$$

Finally, the stiffness factors are:

$$K_A = 409.94 \times \frac{13^2}{5\,264.9} = 1.31588$$

$$K_{AB} = 331.50 \times \frac{13^2}{5\,264.9} = 1.06409$$

and,

$$K_B = 396.50 \times \frac{13^2}{5\,264.9} = 1.27274$$

The stiffness factors having been calculated, it is now a simple matter to apply the equivalent of the Beggs method for finding the fixed-beam end moments. To obtain actual numerical values for the moment at Point A, the deflections must be determined while End A is rotated through a unit angle

TABLE 3.—COMPUTATION OF FIXED-BEAM MOMENTS

Point	C_{AB}					C_{BA}			
	Elastic factor, $\frac{dI}{t^3}$ (1)	Moment (2)	Values of $\frac{M\,dL}{t^3}$ (3)	Values of Σ (from Point B) (4)	Values of $\Sigma\Sigma = C_{AB}$ (from Point A) (5)	Moment (6)	Values of $\frac{M\,dL}{t^3}$ (7)	Values of Σ (from Point A) (8)	Values of $\Sigma\Sigma = C_{BA}$ (from Point A) (9)
				3.5196					
A	0.017	4.6319	0.0787		0.000	3.7456	0.0637		0.000
				3.4409				0.0637	
1	0.067	3.9875	0.2672		3.441	3.1129	0.2086		0.064
				3.1737				0.2723	
2	0.147	3.3431	0.4914		6.615	2.4801	0.3646		0.336
				2.6823				0.6369	
3	0.317	2.6986	0.8555		9.297	1.8474	0.5856		0.973
				1.8268				1.2225	
4	0.657	2.0542	1.3496		11.124	1.2146	0.7980		2.195
				0.4772				2.0205	
5	1.256	1.4098	1.7707		11.601	0.5819	0.7309		4.216
				−1.2935				2.7514	
6	1.846	0.7654	1.4129		10.307	−0.0508	−0.0938		6.967
				−2.7064				2.6576	
7	1.718	0.1209	0.2077		7.601	−0.6836	−1.1744		9.625
				−2.9141				1.4832	
8	1.311	−0.5235	−0.6863		4.687	−1.3163	−1.7257		11.108
				−2.2278				−0.2425	
9	0.742	−1.1679	−0.8666		2.459	−1.9490	−1.4462		10.866
				−1.3612				−1.6887	
10	0.354	−1.8123	−0.6416		1.098	−2.5818	−0.9140		9.177
				−0.7196				−2.6027	
11	0.167	−2.4568	−0.4103		0.378	−3.2145	−0.5368		6.574
				−0.3093				−3.1395	
12	0.078	−3.1012	−0.2419		0.069	−3.8473	−0.3001		3.435
				−0.0674				−3.4396	
B	0.018	−3.7456	−0.0674		0.0016 (Check)	−4.4800	−0.0806		0.0049 (Check)
								−3.5202	

and End B is held fixed, exactly the same as in a mechanical model. Referring to Fig. 10(a), it is evident that this condition can be realized by applying at End A the moment, K_A (= 1.31588), and at End B the moment, K_{AB} (= 1.06409). Since one of the operations will consist of multiplying

the various slopes by dL (see Fig. 7), it will save work to do this in advance, by using the moments, $K_A \times 3.52 = 4.6319$, and $K_{AB} \times 3.52 = 3.7456$. The moments along the beam vary between these two values by thirteen equal increments, so that they are very easily tabulated (see Column (2), Table 3). Multiplying by $\frac{dL}{EI}\left(\text{or, } \frac{dL}{t^3}\right)$ gives the angle at each joint; one summation gives the slope of each section (note that the slope at End A equals 3.52 as a check); and the second summation gives the ordinates of the influence line. A similar operation gives the same data for End B (Columns (6) to (9), Table 3).

The values of the moment increments are:

For Column (2):	For Column (6):
$3.52\ K_A = 4.6319$	$3.52\ K_B = 4.4800$
$3.52\ K_{AB} = 3.7456$	3.7456
8.3775	8.2256
Increment $= \frac{8.3775}{13} = 0.644423$	$\frac{8.2256}{13} = 0.632738$

This same method is well adapted to calculating influence lines for fixed arches, especially if the redundants are taken at the elastic center of the arch. It has been used successfully on unsymmetrical two-span arches, such as those illustrated in Fig. 11(a). In this case the statically determinate base system and the redundant forces, X, were assumed to be as shown in Fig. 11(b). The

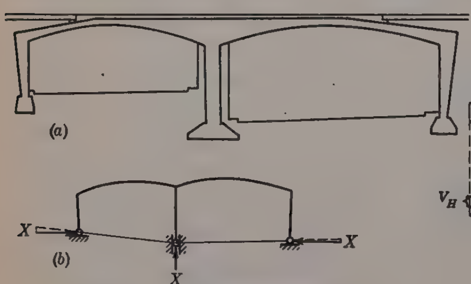

Fig. 11.—A Two-Span Frame.

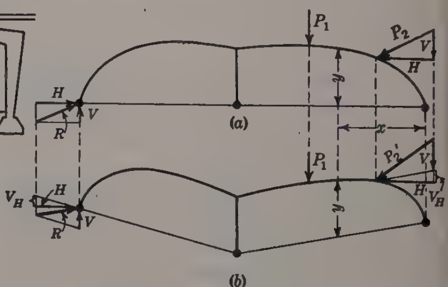

Fig. 12.

three deflections due to each of the three redundants were calculated; then, for each influence line, three simultaneous equations were solved to make the corresponding deflection unity and the other two deflections, zero; and through each of these stages a record was kept of the rotation of the top of the center pier, to serve as a starting point for summing up the changes in direction from point to point.

Truss Deflections

The geometry of trusses, if anything, is even simpler than the geometry of beams, since a few definite members take the place of continually changing

infinitesimal elements. Most of the methods of stress calculation, both graphic and analytic, have some kind of a counterpart in deflection calculation. There is a distinction, however, in the case of resolution of forces or displacements at a joint. From the law of conservation of energy, it will be observed that the relation between displacements is only the reciprocal of the relation between forces, since the product of forces and displacements must be constant. The relation of forces at a joint can be represented by a force triangle or polygon, and the relation between displacements, therefore, automatically cannot. This is the reason that Williot diagrams are inherently more complicated than stress diagrams. For this same reason the dummy load method expedites deflection calculations for single joints or panels.

For complete truss deflections it is preferable to deal with rotations, just as a majority of stress calculations are made by moments. Most of the conditions that can arise, are illustrated by Fig. 13. Consider the deflection at A

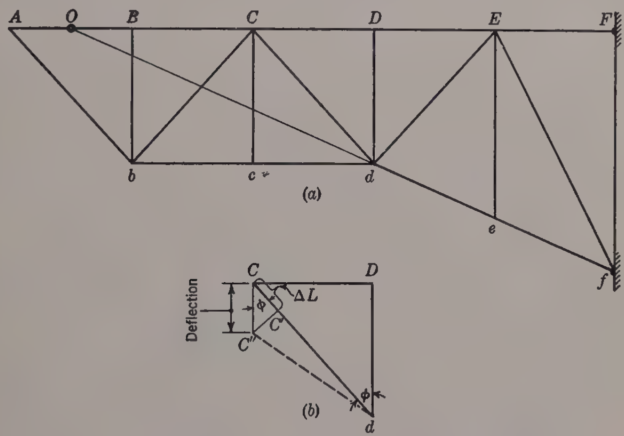

FIG. 13.—DEFLECTION OF A CANTILEVER TRUSS.

due to a change in length of Member CD. The part of the truss bounded by Points d, D, F, and f will remain fixed in place. The part bounded by Points d, C, A, and b will rotate as a unit about Point d. The extent of this rotation is the stretch of Member CD divided by the perpendicular distance from CD to Point d, and the vertical movement of Point A (or any other point on the moving part of the truss) is equal to this rotation times the horizontal distance to Point d. If the member, dE, stretches, the polygon, $dDAb$, will rotate about Point O. This motion can also be treated as a combined translation and rotation about Point D or Point E, expressed entirely in terms of the dimensions of Panel $dDEe$[2].

For a change in length of Member Cd, the rectangle, $dDCc$, is distorted into a parallelogram, and the remainder of the truss is displaced without rotating. The movement is equal to the change of length of Member Cd (ΔL)

[2] *Engineering News-Record,* November 1, 1934, p. 565; and *Proceedings,* Am. Soc. C. E., February, 1936, p. 272.

times $\frac{\overline{dC}}{\overline{dD}}$ as shown in Fig. 13(*b*), because the triangle, $CC'\ C''$, is similar to Triangle dDC, and $\frac{\overline{C\ C''}}{\overline{C\ C}} = \frac{\overline{d\ C}}{\overline{d\ D}}$.

The actual systematic computation of the deflection of a simple truss is shown in Fig. 14 with Table 4. In Fig. 14, the upper values denote changes

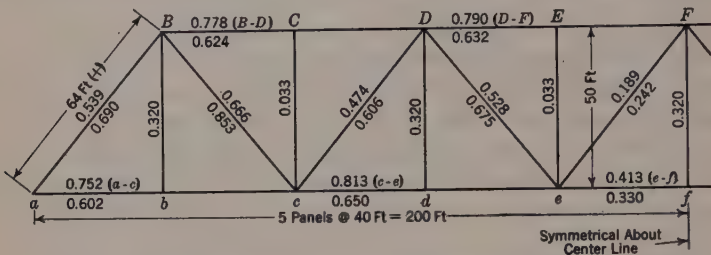

FIG. 14.—DEFLECTION OF A SIMPLE TRUSS

of length, ΔL; and the lower values denote the factor, $\frac{40}{50}\,\Delta L$, for chords and $\frac{64}{50}\,\Delta L$ for diagonals. In a simple truss of this type no algebraic signs are required since the deformations of all members contribute to downward deflection. An alternative method of solving this problem, based on single, instead of double, integration has been presented elsewhere[3] by the writer.

TABLE 4.—DEFLECTIONS OF A SIMPLE TRUSS

Panel Points (see Fig. 14)	CHORDS: Rotation (1)	CHORDS: Slope (2)	Diagonals (3)	Main panel points Column (2) + Column (3) (4)	Verticals (5)	Secondary panel points (6)
a..........				0		
		2.838	0.690			
b..........	0.602			(*B*) 3.528	0.320	(*b*) 3.848
		2.236	0.853			
c..........	0.624			(*c*) 6.617	0.033	(*C*) 6.650
		1.612	0.606			
d..........	0.650			(*D*) 8.835	0.320	(*d*) 9.155
		0.962	0.675			
e..........	0.632			(*e*) 10.472	0.033	(*E*) 10.505
		0.330	0.242			
f..........	0.330			(*F*) 11.044	0.320	(*f*) 11.364

SCOPE OF THE GEOMETRICAL METHOD

Only a few specific applications of the theory of determining deflections by geometry have been presented herein. However, what the writer particularly wishes to emphasize is that a familiarity with the basic rules, comparable with the average engineer's familiarity with forces and moments, will make it possible to improvise methods for solving problems which formerly appeared to be quite difficult.

[3] *Engineering News-Record*, June 7, 1934, p. 746; and *Proceedings*, Am. Soc. C. E., January, 1936, p. 134.

For any one who is interested in acquiring proficiency in this method, a few suggested exercises are as follows:

(1) In a three-hinged arch, write the formula for the deflection at some point on one side due to an angle at some point on the other side. This involves one more operation than a simple beam, just as the simple beam requires one more operation than a cantilever.

(2) Prove the second part of Theorem 3 directly by geometry.

(3) Devise a method for calculating the dead load moments in a two-hinged arch such as in Fig. 8. Then, find what the dead load moments will be if the base is fixed, not by beginning over again, but simply by finding the rotation of the base of the two-hinged arch and then eliminating it, using data on hand as much as possible.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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REPORTS

FILTER SAND FOR WATER PURIFICATION PLANTS

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON FILTERING MATERIALS FOR WATER AND SEWAGE WORKS

The Committee on Filtering Materials for Water and Sewage Works presents herewith its report covering an extended investigation of filter sand for water-works. The report is in two sections; Part I includes certain general statements with reference to filter sand for water-works, and Part II presents the detailed results of experimental studies. These studies, begun in 1926, have been received with much interest by many water-works men, and have inspired a number of other similar investigations. It is significant that as many as sixteen cities were sufficiently interested to set up the experimental test filters and do considerable work in co-operation with the investigations.

The studies have dealt only with sand as a filtering medium, and do not include other less extensively used materials, such as anthracite coal.

PART I.—GENERAL REPORT

Scope of Investigations

Consideration of the problem of filter sand for water-works by the Committee has related mainly to several series of experimental studies under the direction of James W. Armstrong, M. Am. Soc. C. E. These studies may be outlined briefly as follows:

(*a*) During 1927, batteries of seven glass-tube filters were set up by nine co-operating cities using sand supplied by Mr. Armstrong in layers of various sizes and with various depths.[1] The results of these tests indicated that too many variables had been included in both the filter sand depth and in the operating technique.

(*b*) The second series of experiments was begun during 1932, using the same type of equipment, but with a uniform sand depth in all filters and a

NOTE.—Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936. Discussion on this report will be closed in April, 1937, *Proceedings*.

[1] Progress Report of the Committee of the Sanitary Engineering Division on Filtering Materials for Water and Sewage Works, *Proceedings*, Am. Soc. C. E., April, 1928, Society Affairs, p. 243.

uniform laboratory procedure.[2] Mr. Armstrong again furnished all the sand sieved to specified sizes. This series of tests produced many valuable data on lengths of filter runs, wash-water efficiencies, sand rise curves, and some information with reference to floc penetration.

(*c*) During 1934, a series of experiments was run with each glass-tube filter filled with sand of a uniform size. Experiments of this nature were conducted at Baltimore, Md., and Toronto, Ont., Canada. Data on lengths of filter runs, wash-water efficiencies, and penetration of floc were obtained. The equipment used, the laboratory technique, and the results obtained, are described by Mr. Armstrong in Part II.

There has been some discussion, by engineers and others, of the proper procedure for the study of filter sands for water-works. The Committee has been of the opinion that, as a practical matter, a knowledge of filter sand in water filters will be obtained in a larger measure from the results of actual operation of filter plants and experimental filters than from theoretical hydraulic computations. Many variables enter into the operation of a filter bed, and considerable time and study will be required to determine the fundamental basis for studying the separate effect of each of the variables. However, the study of the theoretical and fundamental hydraulic basis for the action of various filter media should be encouraged.

Results of Experimental Work

The detailed results are presented in Part II, and a brief statement only will be given here. The curves showing the relation between length of filter runs and sand sizes indicate quite clearly an increase in length of filter run as the sand size increases. The curves for efficiency of washing also show better results as the size of the top sand increases to a value of about 0.6 mm. The data on penetration of floc obtained by Mr. Armstrong at Baltimore and by Mr. L. F. Allan at Toronto, show a marked increase in penetration with increase in sand size and the need for a relatively deeper sand bed for the sizes of sand.

The later studies by Mr. Armstrong suggest a method for determining the proper depth of sand bed, taking into account the size of sand particles of which it is composed. Further consideration of this may be warranted, particularly with reference to bacterial pollution in the raw water.

Limitations of Experimental Work

The experimental work was limited to small glass-tube filters. It is believed, however, that the results are indicative of those that could be obtained on larger test filters, if they were as well controlled. John R. Baylis, Assoc. M. Am. Soc. C. E., has reported tests on experimental units of various sizes which show that the results obtained from glass-tube filters are reliable as compared with those from test filters with larger surface areas.

The tests cover some variation in the general character of water. There is still some uncertainty as to the application of the test results to all conditions.

[2] "Filter Materials for Water and Sewage Works", *Civil Engineering*, March, 1934, p. 158; also, April, 1935, p. 230.

Extensive tests at Kansas City, Mo., using the highly turbid Missouri River water, give somewhat divergent results that have not as yet been explained satisfactorily.

Any relatively short series of experiments must be considered to produce results which are only indicative of those to be obtained with plant scale operation and with the large number of variables which enter into and cannot be controlled so well in the plant scale operation.

Design Factors

Any consideration of filtering materials should be related to the other elements of the filtration plant. The design of such a plant must take into account several major items, in addition to filtering material, such as: (*a*) Quality of raw water; (*b*) capacities for the plant and its elements; (*c*) type and extent of pre-treatment; (*d*) arrangement and structure of the several plant elements; (*e*) hydraulics of the plant elements; (*f*) wash-water rates and methods of application; and (*g*) miscellaneous or auxiliary equipment and plant fixtures.

In the practice of water purification plant design and maintenance, these various factors have generally been considered of greater complexity and importance than the question of filtering material, and it has been the general practice to standardize such filtering material upon the basis of experience and regardless of the other factors entering into the design of the plant as a whole. Although this practice has generally produced satisfactory results, in some instances the original selection of the filtering material upon such a basis has not proved to be the most desirable.

It appears likely that in the future somewhat greater attention should be given to the selection of the filtering material, together with more careful consideration of other factors entering into filter plant design. Initial or construction costs as well as operation and maintenance costs are important and frequently are the deciding factors in the selection of the filtering materials.

Operating Considerations

The conditions under which a given filter plant must operate and the character of water it must handle are quite variable from day to day, from season to season, and from year to year. The filter plant as a whole, including the filter itself, should be designed to meet these variations and to give a good quality of filter effluent at all times, including periods when the raw or applied water is of inferior quality.

In some cases, the water may be difficult to pre-treat and, at times, with a given filter the pre-treatment may be insufficient, or an unusual variation of quality of the raw water may occur. The filter is the last, and should be a secure, line of defense. Although it is true that under certain conditions the operation of the filter plant may be facilitated during the major portion of the time, by the use of a coarser sand, nevertheless, the designer must give some consideration to those possible infrequent periods of relatively severe loading and should select the material accordingly.

Present Engineering Practice

To obtain data on present practice in filter plant design with reference to sand size and wash-water rates, an inquiry was sent to about twenty-two consulting engineers selected from the "Directory of Engineers", in *Engineering News-Record.* Data from the answers received are summarized in Table 1.

TABLE 1.—Present Practice in Regard to Sand Gradation and Wash-Water Rates

Authority No.:	Sand Grading		Wash-water rate provided for, in inches of rise per minute	Remarks
	Size, in millimeters	Uniformity coefficient		
1.........	0.45–0.55	1.6	36*	Anticipate some variation or reduction in maximum wash-water rates by surface washing, or by some similar development.
2.........	0.35–0.44	1.65	24	....
3.........	0.45–0.55	....	....	Favors air agitation.
4.........	‡	....	30–36	Main objective to obtain efficient 24-in. wash.
5.........	0.55–0.65	1.6	42	Recent design for special conditions.
6.........	0.40–0.45	1.65	36	....
7.........	0.38–0.45	1.5	24	....
8.........	0.40	1.60	36	30%, size 0.50 mm.
9.........	0.40–0.45	1.6	24	Favor other methods of cleaning sand.
10.........	0.40–0.50	1.5	30	Maximum seldom needed.
11.........	0.40–0.45	1.6	18–30	By adjustable pressure valve.
12.........	0.40–0.50	1.7	24†	Maximum determined by 10-lb. pressure on filter.
13.........	0.50–0.55	1.6	42	Finds that provision for liberal wash-water rate does not add materially to cost of plant.
14.........	0.40–0.50	1.6	36	....
15.........	0.40–0.50	1.75	....	....
16.........	0.43–0.47	1.5	32	20 gal per sq ft.
17.........	0.35–0.45	2.5	36	Trend of this office has been toward larger sand size and higher wash-water rates.
18.........	0.40	1.75	24	....
19.........	0.45–0.55	1.5	30	....
20.........	0.40–0.45	1.5	24–27	....

* Maximum.
† Minimum.
‡ Size by sieve numbers and percentages.

With one exception (Authority No. 4) all the engineering offices use the so-called Hazen method for designating sand gradation. The answers in several instances indicated a trend toward a somewhat coarser sand. It is also of interest to compare the present practice of these several engineering firms with the data collected in 1932 by Engene A. Hardin, M. Am. Soc. C. E., covering filter plants in forty-eight cities.[3] The sand size used in the several plants may be summarized as in Table 2.

TABLE 2.—Size of Sand, Plants Built Prior to 1932

Effective Size	Sand Size Equal to or Larger than Given Size	
Sand size, in millimeters	Number of plants	Percentage of plants
0.31	62*	100
0.40	48	77
0.50	11	18
0.60	5	8

* Certain cities reported two or more filter plants, others reported more than one size of sand. The limits of the sand size used were: Average, 0.44; minimum, 0.31; and maximum, 0.62.

[3] *Journal*, Am. Water Works Assoc., Vol. 24, No. 8, p. 1190, August, 1932.

There is also a tendency on the part of some engineers toward a higher rate of wash. Again, the data on wash-water rates collected by Mr. Hardin may be summarized as in Table 3.

TABLE 3.—Wash-Water Rates, Plants Built Prior to 1932

Wash-water rate, in inches rise per minute	Rate Equal to or Greater than Given Rate	
	Number of plants	Percentage of plants
12	61	100
24	41	68
30	12	20
36	4	7
40	0	0

The limits of the wash-water rates reported were: Average, 23; maximum, 38; and minimum, 12.

Designation of Sand Size

There has been considerable discussion of the proper method of designating sand size. Mr. Armstrong believes that the character of filter sand can be adequately revealed only by plotting the results of sieve analysis, and that the top size is more indicative of its character than the Hazen "effective size." Certain engineers have specified sand by actual gradations. Mr. Baylis has recently published a method[4] which seems to fit his experimental data quite well. The so-called "Hazen method"[5], wherein the 10% size is taken as the effective size, has been quite widely used. This method is so deeply rooted that it will be difficult to replace it. Furthermore, it is easy to apply and seems to give fairly reliable results.

Summary

The present trend of engineering practice indicates a range in effective size of sand between the approximate limits of 0.40 mm and 0.55 mm, with some indication of a tendency to adopt the coarser sands where permitted by consideration of the various other factors entering into plant design and maintenance. The results of Mr. Armstrong's experimental work appear, in general, to justify this practice.

While these studies did not embrace the entire range of variables of plant conditions, the coarser sands indicate the following general tendencies: (1) They produce longer filter runs; (2) they produce better "efficiency of wash", as defined in Part II; and (3) they permit greater penetration of floc and thus necessitate a thicker sand layer. A number of additional general conclusions based on the experimental work are set forth in Part II, following the presentation of the test results.

The present status of the art of water filtration includes many unsettled factors. It is not feasible to fix any definite limits for sand size without

[4] *Water Works and Sewerage*, Vol. LXXXI, No. 5, p. 162, May, 1934.

[5] "The Status of Methods for Examining Filter Sand Used in Water Purification Practice", by Paul Hansen, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., April 1927, Society Affairs, p. 168.

giving consideration to the other factors, including costs, that enter into the design and operation of a filtration plant.

The Committee believes the results of the present study will have a decidedly beneficial effect on water filtration practice.

PART II.—DETAILED RESULTS OF EXPERIMENTAL STUDIES

Introduction

In recent years, the difficulty of keeping filter beds clean, the trouble incident to clogging of the sand, and the shortening of filter runs, have been the subject of much discussion. Filter operators and engineers are frequently being confronted with the question of whether or not the size of sand and the depth of bed most generally accepted as standard are, after all, those best suited to filter requirements.

There are too many factors entering into the problem of determining a proper specification for filter sand to arrive at any satisfactory conclusion along purely theoretical lines. Therefore, an effort to answer empirically some of the questions was undertaken through a series of experiments, which were conducted through the co-operation of filter plant operators who had the time, training, and equipment necessary for carrying on such work.

Locations were selected so as to secure a wide range in the character of raw water to be filtered. The following types were used: The highly turbid and alkaline waters of the Middle West; the water from the Great Lakes, which contains great numbers of organisms; and the softer and more highly colored water of the Atlantic seaboard. Experiments covering waters so diverse in character should be more valuable and of greater weight than similar experiments conducted in a single locality.

All experiments of the series begun in 1932 were conducted as far as possible under identical conditions. The co-operating cities used similar sand and worked under the same instructions governing the details of the experiments and the technique to be used.

During 1934, experiments were conducted at Toronto and Baltimore on sand filters of uniform size. From the data secured, formulas were devised for designing a filter bed of unscreened sand. The labor involved was considerable, and a slightly different technique was developed independently in each place. It was thought best not to try to secure exact harmony in methods, especially as the results obtained are in fairly close accord.

Scope of Experiments

Three criteria were selected for judging the suitability of any particular filter sand: (1) Its efficiency in removing suspended matter from the water coming on to the filters; (2) its ability to pass a large volume of clear water between washes; and (3) the ease with which it can be cleaned by the application of wash-water.

In the course of conducting the experiments, a number of lesser but interesting matters were investigated, such as the effect of the viscosity of water upon the washing of the filter bed; and the hydraulic grading of the sand.

Experiments were also run on eight grades of sand of uniform size, in order to determine the minimum depth of bed that will deliver a clear effluent with an 8-ft loss of head.

No effort has been made to determine the laws governing the flow of clean water through clean sand. Although such experiments would be of considerable interest, it is believed that there are too many variable factors entering into the problem of filtration to make such investigations of immediate practical value, and that they would be of little help to any one charged with the responsibility of selecting filter sand or of operating a filter plant.

Selection of Sand Sizes

The eight top sizes used in the 1933 experiments were thought to cover the entire range that could be expected in filter plant operation, the finest being 0.37 mm and the coarsest, 1.75 mm. It was anticipated that the finest size would give runs altogether too short for the best operating conditions, and that the coarsest would probably pass floc very shortly after the filters were put in operation.

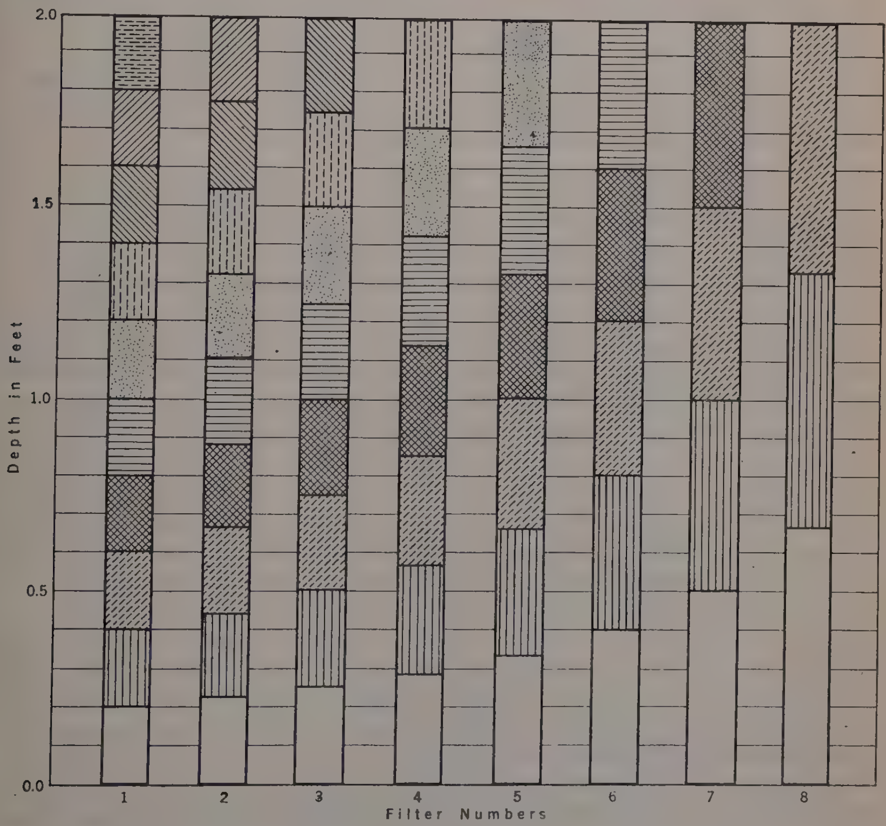

Fig. 1.—Depth and Grading of Sand Used in Experimental Filters.

Throughout these experiments, no effort was made to define sand size in terms of effective size and uniformity coefficient, as the sand was graded arbitrarily in layers of predetermined depth and size. The depth of the different sizes in each is shown in Fig. 1.

The term, top size, was generally employed to designate that of a specific filter, and it was used whenever sand sizes were required for plotting curves. The term, effective size, as ordinarily employed, was not used, because with the sand grading as determined upon for these experiments, it seemed to have no real significance; as with the fine sands, most of the work of filtration occurs in the sizes finer than the 10% size; whereas, with the coarse-grained sands, most of the work is effected in the part of the bed containing materials larger than the 10% size.

TABLE 4.—CHARACTERISTICS OF FILTER SAND, BALTIMORE, MD.

Description	Filter Nos.:							
	1	2	3	4	5	6	7	8
Effective size, in millimeters	0.42	0.49	0.57	0.73	0.94	1.08	1.52	2.06
Uniformity coefficient	2.78	2.58	2.54	2.22	2.15	2.05	1.98	1.57

For the sake of those who are used to thinking in the terms usually employed, the effective sizes and uniformity coefficients of the sands from the eight experimental filters at Baltimore, are given in Table 4. The values were determined in the usual manner.

Basic Considerations and Definitions Applying to the Experiments

Clean Sand.—All filter runs shall be started with clean sand.

Clear Water.—A filter effluent is considered clear when its turbidity does not exceed 0.2 ppm, as determined by a Baylis or similar turbidimeter.

Filter Runs.—A filter run is considered complete when the loss of head reaches 8 ft, the rate of filtration being about 125 000 000 gal per acre per day and the effluent is clear, or when the turbidity of the effluent exceeds 0.2 ppm. The condition reached first terminates the run.

Rate of Filtration.—The maximum rate of filtration shall be 125 000 000 gal per acre per day, and filtration shall be maintained as nearly as possible at that rate.

Properly Treated Water.—A water is properly treated when it contains no colloidal turbidity.

Colloidal Turbidity.—Colloidal particles are those having a diameter less than 500 m mu (millimicrons). Accurate detection of colloidal particles is only possible by the use of the ultra microscope, which utilizes the principle of the Tyndall light beam, converging at a small point and reflecting the illuminated particles. Particles with a diameter of less than 20 m mu will not settle under the influence of gravity.

Laboratory Test.—A laboratory check for determining a properly treated water is to filter a quantity through very fine-grained filter paper. Any turbidity in the filtrate is considered to be of colloidal dimensions.

Bacterial Tests.—Some of the cities were already doing all the bacterial work their laboratories would permit, and others pre-chlorinated their water, making bacterial tests on individual filters useless. In view of these difficulties, and in consideration of the fact that in regular plant operation a sparkling clear filter effluent is usually of good bacterial quality, it was decided not to make any bacterial tests.

Critical Depth.—Critical depth is the term used to designate the maximum depth to which silt will penetrate into a bed of uniformly graded sand up to a loss of head of 8 ft when the filter is delivering a clear effluent, at a rate of about 125 000 000 gal per acre per day. It is defined as that point beyond which the turbidity of water washed from the portion considered is less than 10 ppm, when the volume of wash-water used is in the proportion of 1 liter to each $\frac{1}{2}$ in. of sand depth. The volume of sand thus represented is approximately 19.75 cu cm, and its weight is approximately 30 grams.

Turbidity Factor.—The term, "turbidity factor", was used to designate the product of the turbidity of a quantity of water, in parts per million, multiplied by its volume, in liters. It is used in the formula for determining the efficiency of wash and applies only to the wash-water collected from the primary and secondary washes of the small experimental filters.

Top Size.—Top size is the term applied to the rated size of the sand in the top layer of a filter bed.

Grading of Filter Sand

Although the grading of filter sands shown in Fig. 1 was purely arbitrary, and could not be secured by using any natural sand, a definite method was followed. Each filter contained a total of 24 in. of sand. Ten different sand sizes were used in the experiments. The first filter was filled with ten different layers, each 0.2 ft in depth. In the second filter, the finest size was omitted and the tube was filled with layers of equal depth of each of the nine remaining sizes. In each of the succeeding filters this process was repeated, until in Filter No. 8 there were only three sizes, each 8 in. in depth. This arrangement produced in each filter a progressive grading of sand with sufficient depth in the top layer to remove most of the suspended matter coming on to the filter, except in the coarse-grained filters.

A great quantity of sand was screened through laboratory sieves for the graded sand tests, and it was not re-screened. As a result, some irregularity occurred in the grading used in the various places, which affected to some extent the results of the tests. In the experiments with sand of uniform size, all the sand was carefully re-screened and any fine material that remained in the tube after hydraulic grading, was removed.

Hours of Service

One of the surprising things revealed by the experiments is the fact that Filters Nos. 7 and 8, which had a top sand size of 1.17 and 1.75 mm, respectively, frequently gave long runs without passing floc. The average time of service of Filter No. 8 in all co-operating cities was approximately 180 hr. A study of the curves showing the relation between sand size and hours of

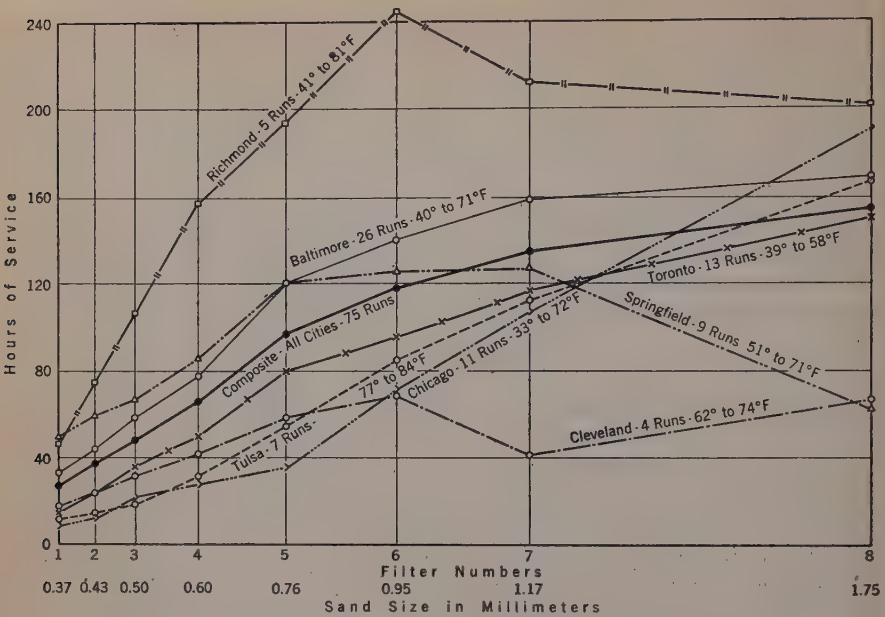

FIG. 2.—RELATION BETWEEN SAND SIZE AND HOURS OF SERVICE, EXPERIMENTAL FILTERS WITH GRADED SAND (COMPOSITE CURVES).

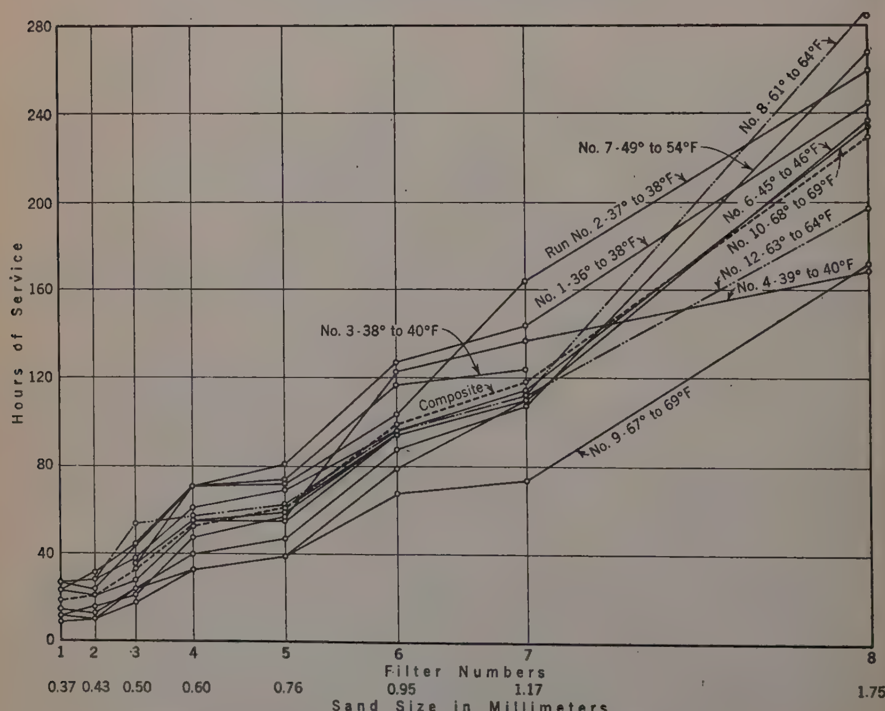

FIG. 3.—RELATION BETWEEN SAND SIZE AND HOURS OF SERVICE, BALTIMORE (MD.) EXPERIMENTAL FILTER, WITH UNIFORM SIZE SAND.

service (Figs. 2, 3, and 4), both those of the composite type, and those of the individual runs of the various cities, showed with a few exceptions an increased length of service as the size of the sand increased. The position of the curves from the different cities varied considerably in the hours of service scale, but when the composite curves of these cities are compared, they show a more uniform trend than would be expected from the individual filter runs.

In Richmond, Va., Chicago, Ill., Tulsa, Okla., and Baltimore, Md., most of the runs of Filters Nos. 1 to 5, inclusive, were terminated by loss of head. In Filters Nos. 6 to 8, inclusive, the runs were more frequently terminated by passing floc than by loss of head. Baltimore had one, Toronto two, and Tulsa five runs of Filter No. 8 that were terminated by reaching an 8-ft. loss of head. The very short runs of Chicago and Cleveland, Ohio, can be accounted for by the presence of large numbers of micro-organisms in the raw water.

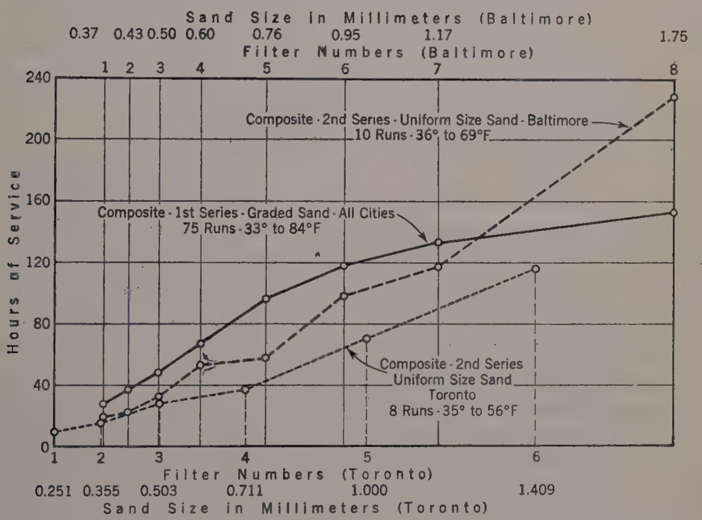

FIG. 4.—RELATION BETWEEN SAND SIZE AND HOURS OF SERVICE: COMPARISON OF COMPOSITE CURVES.

A comparison of the composite hours-of-service curves of the 1933 graded series for all cities and the 1935 uniform series for Baltimore (Fig. 4), shows a surprising similarity of trend and position except for Filter No. 8. A comparison of the graded series with the uniform size series indicates that some of the runs in the graded series were probably terminated before the floc penetrated beyond the top layer. The sand in Filter No. 8 of the graded series was not deep enough, and the runs were generally terminated by the passing of floc, which accounts for the flatness of the slope in the curve. The runs for the Toronto uniform series (Fig. 4), are shorter than those made at Baltimore, but they are comparable with those at other places where the number of micro-organisms is high.

Micro-Organisms

It will be noted from the curves that the lengths of filter runs are very much shorter in some cities than in others. The shortest runs were observed in cities the water supplies of which were taken from the Great Lakes or from large impounding reservoirs. The presence of organisms in sufficient numbers to shorten filter runs materially, occur more frequently in late summer and fall, but organisms sometimes appear in large numbers at other seasons as well. If, in some localities, filters containing sand of a given size show a marked decrease in length of runs, the possibility of trouble with micro-organisms should not be overlooked.

Washing Filters

It is extremely desirable that the filter beds should be kept clean and that there should be as little residual sediment left in the sand bed after washing as possible. In an ordinary filter plant it is not possible to determine how much sediment is actually left in the bed, but it is known that, after a few years, it sometimes becomes necessary to remove, wash, and replace all the filter sand in a plant.

Considerable was learned about the washing of filters by simply observing the behavior of the sand under the influence of wash-water. In the filters containing fine sand, the penetration of floc was very slight. A thin layer of compact sediment forms over the top of the sand surface, which is readily broken under the influence of wash-water. However, instead of being carried out with the ordinary floc, portions of it settle back into the sand bed. The wash-water does not easily clean fine-grained filter beds, and floc will often adhere to the individual grains after being subjected to a velocity sufficient to lift the sand over the gutters in ordinary filters. The floc adhering to the fine grains reduces their specific gravity and, because of the high expansion of the sand, the space between the individual grains is increased to the point where the velocity of the wash-water is not sufficient properly to remove the turbid water, unless it is increased to a rate that would endanger the loss of sand.

There are three reasons why the fine sand is not properly cleaned during the washing period: The fineness of the small grains makes it possible for the floc to envelop them entirely, the irregularity of their surfaces makes it difficult to remove the floc after it has become attached and at ordinary rates the sand expansion is great enough to prevent the striking together of the grains with sufficient force to loosen the floc, and the mass of each grain is so small that there is little effect when they do strike. Even in the laboratory, where the sand is put in a beaker with water and stirred with a glass rod, it is very difficult to remove all the floc.

The behavior of the filters containing coarse sand is very different. The filter runs are materially longer, and produce an accumulation of loose floc on top of the filter that does not, except after very long runs, form into a tough cake as it does in the fine sand filter. Eventually, the floc in the filters with coarse sand will penetrate to the bottom of the sand layer, but it takes a

surprisingly long time. Upon washing, the mud cake on top of the filter is generally easy to break, and particles do not often settle into the filter bed. A higher velocity of wash-water is required to lift the coarse sand. While in suspension the particles do not grade hydraulically, in any strict sense of the word; the grains of different sizes are continually rising and falling; and the contacts thus made loosen the floc and permit it to pass out of the filters after a relatively short wash. The water above the sand is left clear, and when the sand settles back into place, it is with only a small amount of floc adhering to it.

From these observations, it appears that, in order to keep a filter bed clean, a certain minimum velocity of wash-water is necessary. If this velocity is high enough to agitate the sand properly, the cleansing effect of the water will be greatest with that sand which has the lowest expansion.

Two-Period Washing.—Observation of the small filters during the washing period indicated that filters can be kept cleaner by applying the wash-water at two separate rates. First, the sand should be lifted just high enough to agitate it and loosen any adhering silt; then it should be expanded a further amount by a rate sufficiently high to clear the filter bed of any loosened sediment. This observation is apparently borne out by a limited practical experience on large filters.

Efficiency of Wash

A simple formula has been devised to indicate the efficiency of a filter wash. The wash is divided into two parts: Primary and secondary. The primary wash corresponds to that ordinarily performed in a filter plant, and the secondary wash consists of washing out all the sediment that remains in the filter bed after the first operation is completed. The efficiency of a filter wash is the percentage of total sediment deposited in the sand during a filter run that is removed in the primary wash.

The procedure in washing is as follows: All the water used in the regular or primary wash is collected in a bucket. After the primary wash is completed, the water is drawn down within a few inches of the sand surface, and sufficient wash-water is then admitted just to lift the sand. A wire grid, bent like a potatoe masher and secured at right angles to the axis of a light steel rod, is then worked up and down through the sand, loosening all the suspended matter from the grains and breaking up any small mud particles remaining in the filter bed. As soon as the sediment is all loosened, the filter is again washed at a higher rate and all the effluent is caught in a bucket. The volume and turbidity of both primary and secondary washes are then carefully determined. The efficiency is then computed from the formula,

$$e = \frac{100\, Z_p}{Z_p + Z_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which e = percentage of efficiency of the filter wash; Z_p = turbidity factor of the primary wash = $T_p\, Q_p$; Z_s = turbidity factor of the secondary wash = $T_s\, Q_s$; T = the average turbidity of the total volume of wash-water, in

parts per million; and Q = total quantity of wash-water, in liters, recorded to the nearest one-hundredth. The subscripts, p and s, refer, respectively, to primary and secondary wash.

Four factors affect the efficiency of a filter wash: The velocity of water in passing upward through the sand bed; the duration of the wash; the size of the sand grains; and the temperature of the water.

The data on efficiency of wash received from each co-operating city were plotted. Composite curves from seven cities are shown in Fig. 5, and those from a single city, Tulsa, are presented in Fig. 6. These experiments were performed on the graded filters, and in each case the sand size shown is the size of the top layer.

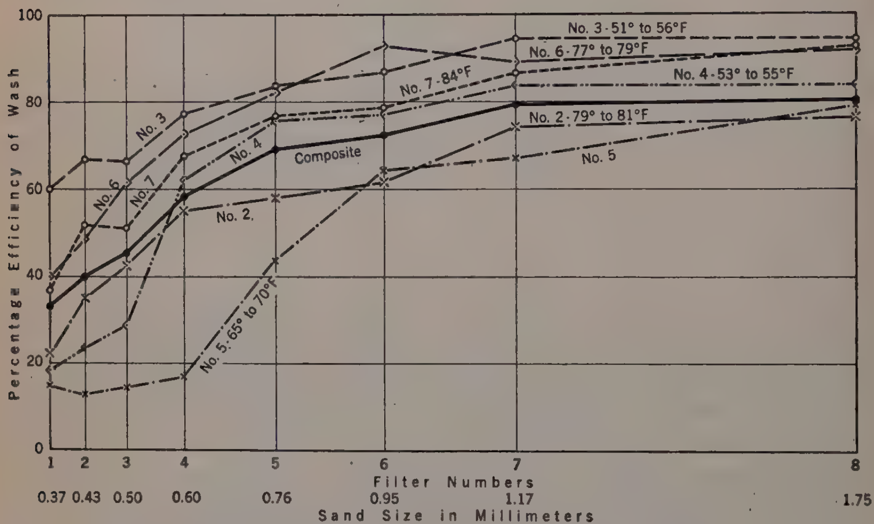

FIG. 5.—EFFICIENCY OF WASH, EXPERIMENTAL FILTERS AT TULSA, OKLA.

These curves show a much wider variation than those for the hours of service. The greater irregularity can probably be accounted for in the number of variables affecting the results—the velocity and the viscosity of the applied water, the duration of the wash, the character of the silt deposited in the sand bed, micro-organisms, and the personal equation.

The composite curves at first sight seemed to have little value, but it is believed that they do represent what might be considered a normal trend and that they smooth out the irregularities due to the number of variables that seemed impossible to evaluate in studying the curves from the various cities. Much of the apparent inconsistency in the observed data would disappear if all the factors involved could be appraised correctly.

A comparison of the composite curve of the efficiency of wash and that for the hours of service of the various cities shows that as the sand increases in size, the value of both increases up to and including Filter No. 7, and, in a number of instances, there is an increase in value for Filter No. 8.

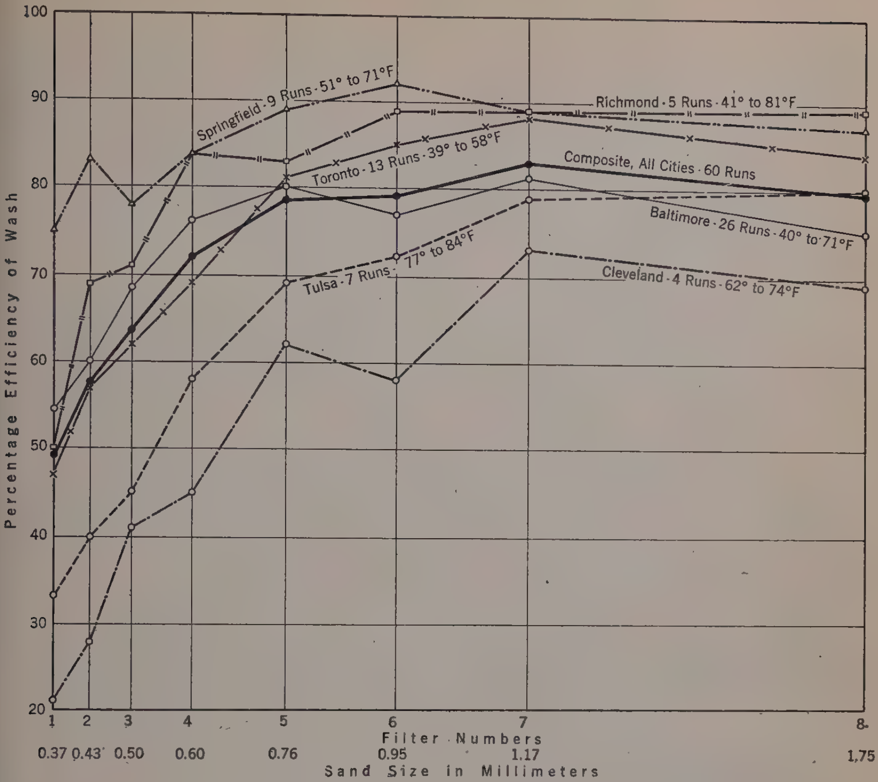

FIG. 6.—EFFICIENCY OF WASH (COMPOSITE CURVES).

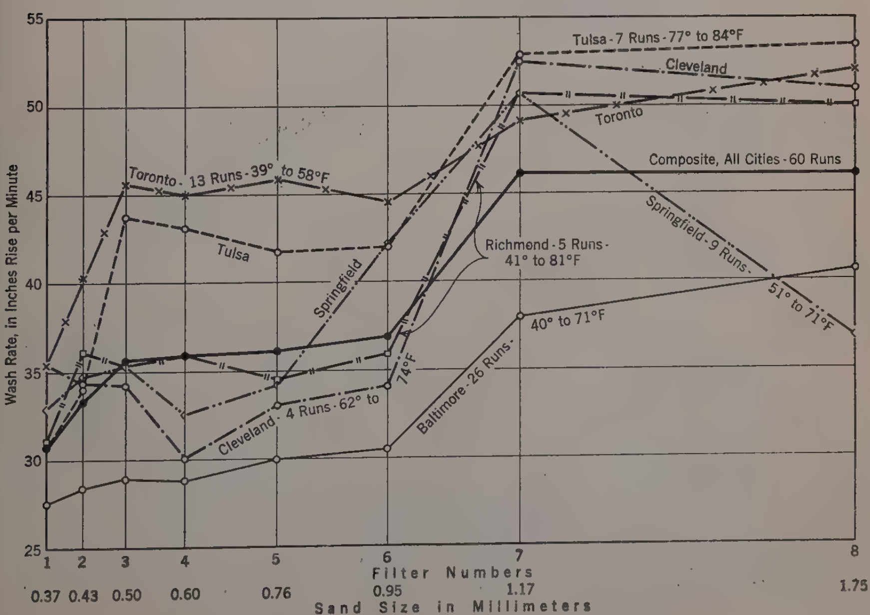

FIG. 7.—AVERAGE RATES OF APPLYING WASH WATER.

If the curves of Fig. 6 are studied in connection with those of Fig. 7, which show the average rate of applying wash-water, it will be noted, especially with the finer sands, that the cities using the higher rates of wash did not hold the highest place in the scale of efficiency. The curves also indicate that coarse sands are more efficiently washed than fine sands, and that the rate of applying wash-water should be increased as the sand size increases. There appears to be an optimum rate for washing, and to increase the rate beyond that point only results in a decreased efficiency. The optimum rate is lower when the water is cold.

It is also interesting to note that, although a lower average rate of applying wash-water was used in Baltimore than in any other city, yet the efficiency was better in the filters containing the finer sizes of sand, than in most places where a much higher rate of wash was used. With the filters containing the coarser sand, the rate of wash was too low at Baltimore for the best efficiencies.

Experiments with Sand of Uniform Size

The experiments with sand of uniform size were undertaken in the hope of securing basic information regarding each grade of sand, that would be valuable in designing filter beds and in interpreting the results of other experiments. Baltimore and Toronto were the only cities to co-operate in this series.

Twelve runs for determining the critical depth were made at Baltimore and four at Toronto. The technique varied somewhat, with the result that the two cities used slightly different methods of reporting their experiments.

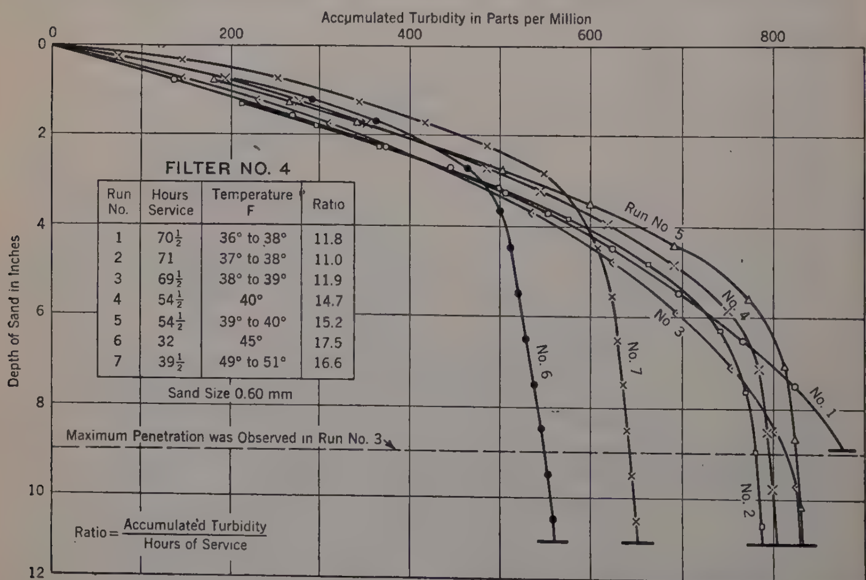

FIG. 8.—TYPICAL ACCUMULATED TURBIDITY CURVES, BALTIMORE EXPERIMENTAL FILTERS, WITH UNIFORM SIZE SAND.

Experimenters in Toronto recorded turbidities as "turbidimilligrams" (see Appendix), In Baltimore, the term, "accumulated turbidity", was used. The

work done in the two cities is comparable, but, in Baltimore, the results were plotted as accumulated turbidities; that is, the turbidity of each section was added to the previous one (see Fig. 8). By this method any point on the curve represents the total quantity of sediment in the bed between the surface and the depth at which the point is taken, and the slope of the curve indicates the intensity of the floc penetration in the sand bed.

Penetration Curves.—For the critical depth experiments, sand was placed in the tubes to a depth estimated to be greater than critical, and the filter runs were begun and continued until terminated by an 8-ft loss of head. (If the sand depth originally provided was not sufficient to give a clear effluent at this head loss, it was increased before the next run was made.) The maximum penetration of floc was then determined in the manner described in the Appendix.

The critical depth for each filter, used in plotting Fig. 9, was the maximum value obtained in any run on that filter. The maximum depth of penetration was in almost all instances well below the points where the cumulative curves take a sharp downward turn.

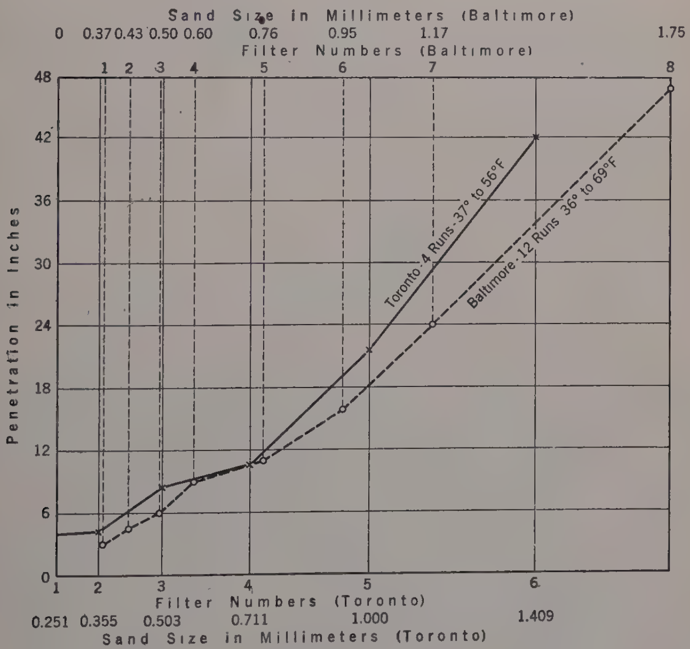

FIG. 9.—MAXIMUM PENETRATION OF FLOC IN EXPERIMENTAL FILTERS.

The penetration curves fell into two distinct groups: Those for runs made when the temperature of the water was below 45° F, and those for runs at 45° F and above (Fig. 10). The fact that, with warmer water, the runs were shorter, and that the silt did not penetrate as deeply into the sand bed, can be accounted for partly by the presence of a large quantity of organic matter in the water at such times. The deeper penetration occurred over the same

temperature range that affects the chemical treatment of water the most; namely, that in which the viscosity of the water is the greatest. These facts suggest the possibility of using a shallower sand bed in climates where the

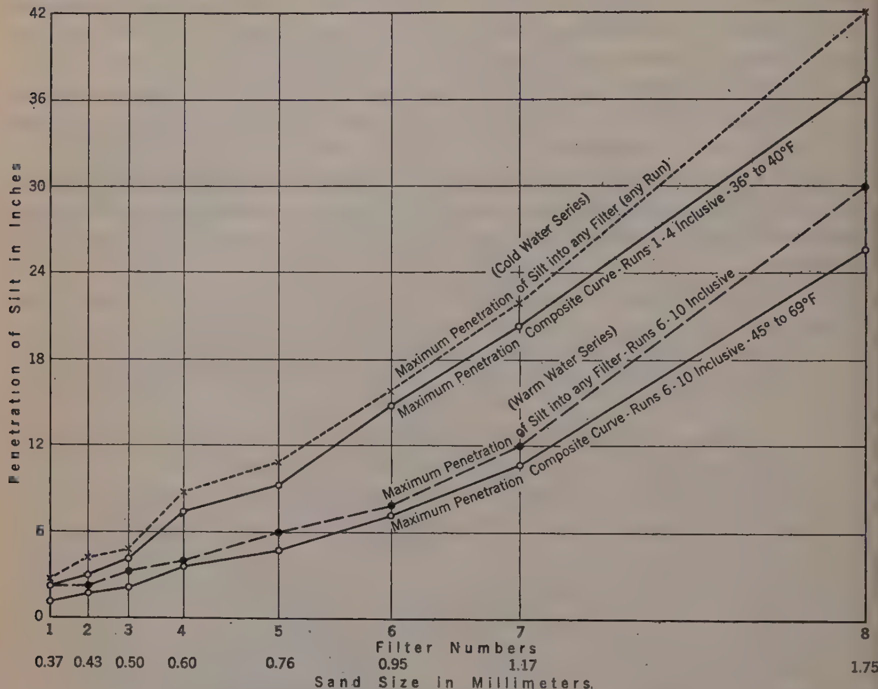

FIG. 10.—PENETRATION OF FLOC IN EXPERIMENTAL FILTERS, SHOWING VARIATION WITH TEMPERATURE.

temperature of the water does not fall below 45° F. Evidence also points strongly to the probability that the maximum depth of penetration, or the critical depth of any grade of sand of uniform size, is proportional to the square of the diameter of the average size of the sand grain (see Table 5).

Comparison of Uniform and Graded Sand Filters.—Curves for sand filters of uniform size and those for graded sand filters revealed the same general characteristics; that is, that the hours of service and the efficiency of wash increase as the sand size increases. These characteristics are more marked, and there is less variation from the mean, with sand filters of uniform size. The greater uniformity of slope of the various curves would indicate that filters composed of sand of a single size would be more dependable than graded sand filters.

DESIGNING A FILTER BED

In submitting data for designing a filter bed, it is with the belief that local sands can often be used without screening. Some sands, however, on account of their peculiar grading, are not suitable for filter purposes unless combined with other sands. In general, the various sizes or grades of sand

particles may be divided into three groups: (1) Grades too fine for filter purposes; (2) grades suitable for the filtering medium; and (3) grades too coarse for filter purposes, but valuable as a supporting medium. After making a sieve analysis, the first step in designing a filter bed is to select the proper limits for each of these groups.

As it does not seem possible to establish, arbitrarily, a rigid standard of procedure, it would be well, before deciding upon the dividing line between Group (1) and Group (2), to give special consideration to the effect of the choice upon the following: The length of filter runs; the cost of wash-water; the efficiency of filter wash; the cost of removing the fine sand; and the advisability of adding a small percentage of specially graded sand.

TABLE 5.—RELATION BETWEEN SAND SIZE AND CRITICAL DEPTH

Filter No.	Sand size*, in millimeters	Square of sand size	Average penetration, in inches	Ratio of average penetration to sand size squared	Filter No.	Sand size*, in millimeters	Square of sand size	Average penetration, in inches	Ratio of average penetration to sand size squared
BALTIMORE, MARYLAND (AVERAGE OF TWELVE RUNS)					TORONTO, ONT., CANADA (AVERAGE OF FOUR RUNS)				
1.........	0.37	0.1369	2.00	14.61	1........	0.251	0.063001	2.52	40.00
2.........	0.43	0.1849	2.57	13.90	2........	0.355	0.126025	3.20	25.39
3.........	0.50	0.2500	3.50	14.00	2........	0.503	0.253009	5.72	22.61
4.........	0.60	0.3600	5.62	15.61	4........	0.711	0.505521	8.48	16.76
5.........	0.75	0.5625	7.17	12.74	5........	1.000	1.000000	17.92	17.92
6.........	0.95	0.9025	11.44	12.67	6........	1.409	1.985281	37.68	18.97
7.........	1.17	1.3689	15.83	11.56	....	......	......	....	....
8.........	1.75	3.0625	32.00	10.45	....	......	......	....	....
Average ratio....	......	......	....	13.19	Average ratio....	......	......	....	23.61
Average, omitting Filter No. 8...	......	......	....	13.58	Average, omitting Filter No. 1...	......	......	....	20.34

* Average of size passing and size retained.

The following method is based on a formula suggested by Mr. Allan, of Toronto, for determining the depth of a filter bed when a sieve analysis of the sand under consideration, and the critical depths of its various grades, are known. It is based on the assumption that the "depth" of each grade of sand in the completed filter is proportional to the percentage by weight of that grade in the given sand, and that the filtering ability of each grade is inversely proportional to its critical depth. Let:

$c_1; c_2; c_3;$.................. = the critical depth, in inches, of each grade of sand as determined by experiment.

$p_1; p_2; p_3;$.................. = the percentage, by weight, of each grade of sand under consideration as determined by sieve analysis.

$p_t = p_1 + p_2 +;$............ = the total percentage of sand, by weight, as shown by sieve analysis, of those sizes that are considered satisfactory for the filtering medium.

$p_{c1}; p_{c2}; \ldots = \frac{100\, p_1}{p_t}; \frac{100\, p_2}{p_t}$ = the corrected percentages of each grade of sand required to build the new filtering medium.

d_m .... = the computed minimum depth of sand, in inches, to be placed finally in the bed as a filtering medium.

$a_1; a_2; \ldots$ .... = the filtering ability of each grade of sand comprising the filtering medium, as a percentage of the filtering ability of the complete bed.

The filtering abilities of all grades of sand, up to their respective critical depths, are equal. In the new bed the filtering medium, d_m, must have the same filtering value. The filtering ability of any grade in the filter, in proportion to the total required, is, therefore, equal to the depth of that grade divided by its critical depth: $a_1 = \frac{p_{c1}\, d_m}{c_1}$; $a_2 = \frac{p_{c2}\, d_m}{c_2}$. Also, $a_1 + a_2 + \ldots\ldots = \frac{p_{c1}\, d_m}{c_1} + \frac{p_{c2}\, d_m}{c_2} + \ldots\ldots = 100$ per cent. Whence,

$$d_m = \frac{100}{\frac{p_{c1}}{c_1} + \frac{p_{c2}}{c_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If d_f = a depth of sand, in inches, to be added to d_m as a factor of safety, d_t = total depth of sand to be placed originally in the filter; p_f = percentage, by weight, as shown by sieve analysis, of sand too fine for filter purposes; and p_s = percentage, by weight, as shown by sieve analysis, of coarse sand, which will form the supporting medium. Then,

$$d_t = \frac{(d_m + d_f)\, 100}{p_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The depth, in inches, of fine sand to be removed from the filter $= p_f\, d_t$, and the depth of the supporting medium $= p_s\, d_t$.

For an example of the application of these formulas, see Table 6. The sieve analysis used in the example, is that of a sand submitted for filter purposes, and it is assumed that the portion finer than 0.60 mm should be removed. The factor of safety is assumed as one-fourth the depth of the filtering medium.

From Table 6, $\Sigma a = 7.56\, d_m = 100\%$; $d_m = \frac{100}{7.56} = 13.2$ in. Since $d_f = 0.25\, d_m$, $d_t = 21.8$ in.; $p_f\, p_t = 2.96$ in.; and $p_s\, p_t = 2.35$ in.

Should the sand from a filter be replaced by a coarser grade of less depth, it is possible on account of the increased free-board, that there would not be sufficient velocity of wash water properly to remove the silt washed from the sand, unless its rate of application were increased.

TABLE 6.—PROPORTIONING THE DEPTH OF SAND IN A FILTER BED FROM A GIVEN SIEVE ANALYSIS

Sieve No.* (1)	Grade No.† (2)	Size of sand, in millimeters (3)	c_1; c_2;‡ (4)	Sieve Analysis, Percentage Retained, by Weight: p_f (5)	$p_t = p_1 + p_2 +$ (6)	s (7)	P_{c1}; P_{c2};.. (8)	a_1; a_2;.. (9)
Pan:	....	....	....	0.86	....	.....	....	....
100..........	....	....	....	0.04	....	.....	....	....
80..........	....	....	....	0.04	....	.....	....	....
60..........	....	....	....	0.12	....	.....	....	....
50..........	1	0.37	3.0	0.36	....	.....	....	....
45..........	2	0.43	4.5	2.08	....	.....	....	....
40..........	3	0.50	6.0	10.06	....	.....	....	....
35..........	4	0.60	9.0	....	5.36	.....	7.08	0.79 d_m
30..........	5	0.75	11.0	....	40.06	.....	52.97	4.81 d_m
24..........	6	0.95	16.0	....	17.46	.....	23.08	1.44 d_m
20..........	7	1.17	24.0	....	5.90	.....	7.80	0.33 d_m
16..........	8	1.75	47.0	....	6.86	.....	9.07	0.19 d_m
10..........	....	2.50	....	....	....	5.42	....	....
8..........	....	3.75	....	....	....	5.38	....	....
Totals........	....	....	....	13.56	75.64	10.80	100.00	7.56 d_m

* Howard and Morse sieve. † Sand used in experiments. ‡ From Fig. 9.

TEST RUNS TO CHECK EXAMPLE

In order to test the formula, a filter bed was graded in accordance with the example, and an extra depth of 25% was added as a factor of safety. The test resulted in a filter run of 43 hr and a maximum penetration of 7 in., or less than one-half the depth of the sand bed. In this case it is evident that a 25% increase in depth gave more than 25% additional filtering ability, and prevented a close check on the example.

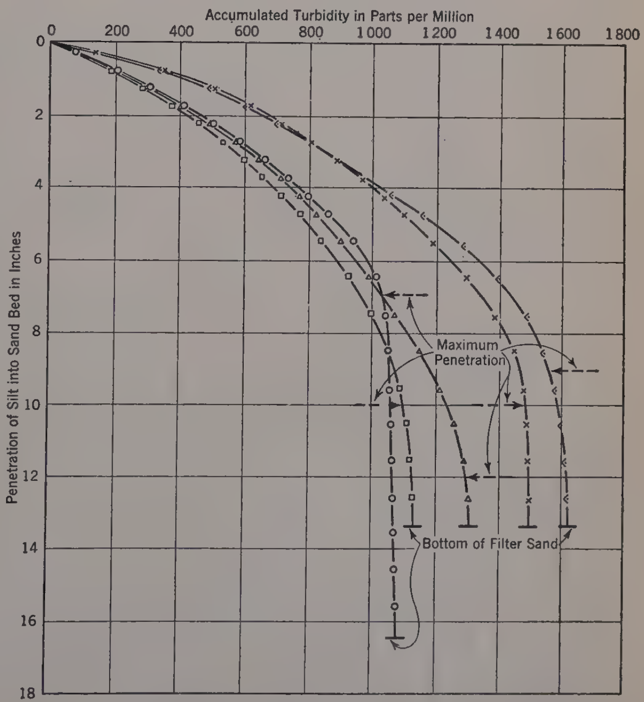

FIG. 11.—TEST RUNS TO CHECK FILTER DESIGN FORMULA.

The sand was then removed and graded exactly in accordance with the example, without any extra depth as a factor of safety. Four

runs were then made without having the silt penetrate to the bottom of the filter bed in any case (see Fig. 11).

More tests should be run to check the reliability of the formula, but if those cited previously can be considered as indicative of the general performance it would appear that a rational method has at last been developed for determining the proper depth of a filter bed.

Miscellaneous Remarks

No investigation was made of the effects of the shape of sand upon the filtering or washing processes, but it is believed that round, smooth, sand grains are more desirable than irregular or angular ones, as they are more readily cleaned of adhering silt, and the removal of an additional quantity of silt would result in better filter efficiency. This observation is based on the fact that smooth gravel is more easily cleaned than rough gravel. It is also believed that thin, flat, sand grains would more readily become compacted and would tend to build up loss of head faster than smooth grains.

Theoretical Discussion.—Theoretical discussions of the laws governing the flow of clean water through sand have created quite an interest, and those who care to investigate the matter are referred to a paper[6], by Roberts Hulbert and Douglas J. Feben, on the "Hydraulics of Rapid Filter Sand," and one by Gordon M. Fair, M. Am. Soc. C. E., and Loranus P. Hatch, Jun. Am. Soc. C. E., entitled "Fundamental Factors Governing the Stream Flow of Water through Sand."[7] An interesting thesis has been written by Messrs. H. H. Black and C. N. Stutz, of the University of Illinois, on "Physical Factors Affecting the Hydraulic Properties of Rapid Filter Sand" (1933).

Supporting Data.—Only a small portion of the data collected during the experiments could be embodied in this report, but it is believed that enough has been submitted to show the methods that were used and the results obtained. The total mass of essential data has been collected in several volumes and placed in Engineering Societies Library, in New York, N. Y., where it is available to any one interested.

Summary of Results of Experiments

Effluent.—A good effluent can be secured with any size of sand, provided the filter bed is of a proper depth.

Depth of Filter Bed.—The depth of a filter bed should be increased as the sand size increases. For sand of uniform size the critical depth, which is roughly proportional to the square of the diameter of the average size of the sand grains, can be used as a guide. For a graded sand, the depth of a filter bed may be designed correctly if a sieve analysis of the proposed sand and the critical depth of each grade are known.

Hours of Service.—The number of hours a properly graded filter can be run between washings, increases as the size of the sand in the filter bed increases.

[6] *Journal,* Am. Water Works Assoc., Vol. 25, January, 1933.

[7] *Loc. cit.,* November, 1933.

Efficiency of Wash.—The efficiency of a filter wash for any given grade of sand, is primarily dependent upon the rate at which the water is applied. It is also affected by the viscosity of the water, and, to a slight extent, by the duration of the wash. Better efficiencies are secured with coarse sands.

Washing Rate.—The rate of applying wash-water should be increased as the sand size of the filter bed and the temperature of the water increases. If wash-water is applied at too high a rate, the efficiency of the wash is decreased. With a graded sand, good results can be obtained with rates ranging from a rise to 24 in. per min for fine sands; and to 30 in. for sand of about 0.6 mm in diameter. With sands having a top size of from 0.76 mm to 1.75 mm, the rate should be stepped up as the size increases to a maximum vertical rise per minute of between 40 and 50 in. A few tests on sand of uniform size indicate, especially with the coarser grades, that lower rates can be used satisfactorily as the uniformity coefficient approaches unity.

Sand Size.—The experiments seem to indicate that sand beds with top sizes ranging in diameter from 0.6 mm to 1.00 mm will give very satisfactory results; that sizes of 0.5 mm and less give short runs, require frequent washing, and are difficult to keep clean. Excellent results can be secured with graded sands having a top size of from 1.00 to 1.75 mm in diameter, but as the upper limit of size is approached, the depth of bed and rate of wash should be increased beyond those ordinarily used.

Uniform Size of Sand.—If the findings of a limited number of tests can be accepted, an ideal filter bed should be composed of sand of uniform size. There seems to be greater uniformity in the hours of service and in efficiency of wash for any grade of sand, if it is of one size only. Coarse-grained filters can be satisfactorily washed at a lower rate than graded filter beds the top size of which is of the same diameter as the uniform bed.

General Statements.—This summary is presented with no thought of finality, especially where figures are concerned. It is believed, however, that the limits given are confirmed by the experiments, and that the information gained will be a helpful guide in determining the size and depth of a sand bed. The character of water, and other local conditions, vary so greatly in different parts of the country, that it is not possible to set up any standard that would be generally acceptable.

Cost, in most instances, will probably be the deciding factor in the selection of a sand. When this is the case, the possibility of using local sand should be carefully considered. In all cases a balance should be preserved between the cost of an ideal sand bed and the handicap that will result from a compromise. Fine sands mean frequent washing and more trouble in the maintenance of a filter bed, while coarse sands may mean deeper filters and larger piping and valves.

Acknowledgments

The following acknowledgments are made with a realization of the impossibility of giving full credit to every one connected with the experiments. Only those with whom correspondence was conducted, or who are known

to have been in direct charge of the work, are mentioned. Much faithful service was rendered by those engaged in routine work, whose only reward lies in the knowledge of the faithful performance of duty. Some of those named worked only on the earlier experiments, and although the results they obtained could not be used in this report, their work was of much value in preparing the way for those who followed. Those in responsible or direct charge of experiments in the various cities were: In Baltimore, Md., George C. Dobler, Senior Analytical Chemist, and George B. McCall, Junior Chemist, who conducted most of the experimental work, and Edward S. Hopkins, Principal Sanitary Chemist, who prepared the bibliography (not published); in Chicago, Ill., John R. Baylis, Assoc. M. Am. Soc. C. E., Physical Chemist, Bureau of Engineering; in Cincinnati, Ohio, Clarence Bahlman, Water Purification Supervisor; in Cleveland, Ohio, J. W. Ellms, M. Am. Soc. C. E., Engineer of Water Purification and Sewage Disposal, and L. A. Marshall, Superintendent of Division Filtration Plant; in Denver, Colo., D. D. Gross, M. Am. Soc. C. E., Chief Engineer, Board of Water Commissioners, and O. J. Ripple, Superintendent of North Side Filter Plant; in Kansas City, Mo., T. D. Samuel, Jr., M. Am. Soc. C. E., Chief Engineer and Superintendent of Water Department, and Dr. G. F. Gilkerson, Chief Chemist, Filteration Plant; in Omaha, Nebr., Theodore A. Leisen, M. Am. Soc. C. E., General Manager, Metropolitan Utilities District; in Providence, R. I., Elwood L. Bean, Chemist, Scituate Reservoir Division; in Racine, Wis., Walter A. Peirce, M. Am. Soc. C. E., Manager of Water Department; in Richmond, Va., M. C. Smith, Engineer in Charge of Water and Electricity; in Sacramento, Calif., R. A. Stevenson, Superintendent of Filtration; in Springfield, Ill., C. H. Spaulding, Superintendent of Water Purification, Municipal Water Purification Plant; in St. Louis, Mo., A. V. Graf, Chief Chemical Engineer, Water Purification Plant; in Chesterfield, Mo., A. G. Nolte, Chief Chemical Engineer, Chain of Rocks Plant, and J. C. Pritchard, M. Am. Soc. C. E., Director of Public Utilities; in Toronto, Ont., Canada, L. F. Allan, Superintendent of Filtration Plant; in Tulsa, Okla., A. B. Jewell, City Chemist, City Health Department; and in Washington, D. C., E. D. Hardy, Engineer in Charge of Water-Works Extension, and C. J. Lauter, Chief Chemist, Dalecarlia Filter Plant. Mr. Armstrong, of the Committee, directed and correlated the experimental work, and personally prepared Part II of this report, and the Appendix.

Respectfully submitted,

W. E. Stanley, *Chairman,*
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January, 1936.

APPENDIX

Early Experiments

Before starting the experiments in 1927, the Committee sent a questionnaire to all State Sanitary Engineers to learn, if possible, the reasons governing their requirements relating to the size and depth of filter sand. The answers indicated an effective size in use ranging from 0.2 to 0.65 mm in diameter, and depths of filter beds from 24 to 36 in. Apparently, sand was selected as a matter of custom or simply on the grounds of personal opinion, and there was evidently little information that would permit a rational selection to be made.

After studying the results of the questionnaire, it was decided to ask a number of cities to co-operate in conducting experiments with small glass filters, and nine of them actually built and operated a battery of seven small glass filters with an inside diameter at approximately $1\frac{3}{4}$ in. One of the units was used for comparing results with plant performance and was filled with sand from one of the large filters.

As the experiments proceeded, many questions arose regarding the technique. It was found difficult to secure uniform results, largely on account of the varying conditions under which the co-operating cities found it necessary to operate the small filters. The time necessary to conduct the experiments, especially as an effort was made to determine the bacterial quality of the filter effluent, was more than some cities could devote to the work.

After comparing the reports of all the co-operating cities and studying the results of their experiments, there was found to be more uniformity than was at first apparent, but there were too many variables to make the comparisons entirely satisfactory. The tests seemed to indicate that the coarser sand gave longer filter runs with an equally good effluent, but the fact that the depth of the top layer as well as the total depth of the sand bed varied, made it difficult to evaluate the effect of size of sand on the length of run. All the top layers were carefully screened to as nearly the specified size as possible, but as it was believed that most of the work of the filters would be done in the top layer, the remainder of the bed was looked upon less as a filter than as a supporting medium.

The necessity for a carefully planned technique for regulating every stage of the experiments became more apparent as time went on, but it was not until the spring of 1932 that there was sufficient agreement among those co-operating in the experiments to permit the drafting of a set of instructions that would be generally acceptable. From the earlier experiments much was learned that proved helpful in preparing for and conducting the later work. A description of the 1927 series of tests as conducted at Denver, Colo., was written[8] by Mr. O. J. Ripple; the St. Louis experiments were described[9] by Mr. August C. Nolte. Mr. L. A. Marshall discussed the experiments in Cleveland, in a paper presented before the Ohio Conference of Water Purifica-

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[8] *Journal,* Am. Water Works Assoc., Vol. 23, p. 1317.

[9] *Loc. cit.,* p. 1311.

tion Plant Operators, in October, 1932; and Mr. L. F. Allan published a paper[10] on the experiments as conducted at Toronto, including those on sand of uniform size.

Later Experiments on Graded Sand Beds

When the second set of experiments was undertaken, the program was simplified and the number of objectives was reduced, but there still remained some difficulties to be overcome in the matter of technique. A satisfactory set of instructions was finally drawn up in the spring of 1932. The remainder of this Appendix relates solely to these later experiments.

In the instructions sent to the co-operating cities, the different items to be investigated were grouped as follows:

Primary Data to Be Secured.—Filtration: To determine for the various sizes of sand:

(1) The ability of the filter bed to prevent the passage of floc;
(2) The length of run;
(3) The volume of water filtered during the run; and
(4) The effect of temperature on length of run.

Washing: To determine for the various sizes of sand:

(5) The best rate of applying wash-water to clean the filter bed effectively; and
(6) The effect of temperature on washing rates.

Secondary Data to Be Secured.—

(7) The velocity of water required for lifting sand of various sizes by carefully measuring small increments, to a sufficient height above the surface of the filter bed to permit the plotting of sand rise curves;
(8) The depth of floc penetration and the volume of floc retained in filter beds during the period between washings;
(9) The extent of hydraulic grading of sand under the action of wash-water;
(10) The effect of temperature and viscosity of water on sand rise; and
(11) The effect of any unusual local conditions that may be due to organic content of water, alkalinity, turbidity, color, iron, or manganese.

In inviting cities to co-operate with the Committee in conducting the experiments, an effort was made to secure those in which the water supply would be representative of different types. Two separate sets of experiments were undertaken, and sixteen cities went to the expense and trouble of building batteries of small filters and carried on the experiments for a more or less extended period. There were other cities that wanted to undertake the

[10] *Canadian Engineer*, Vol. 66 (1934), No. 21, p. 9.

experiments, but did not have sufficient personnel to justify their doing so. Some of the cities that undertook the task, found that it was not possible to carry on the experiments without employing additional help; others after making a start realized that they had no employees qualified for the undertaking; and every city found difficulty at times in conducting the experiments, due to vacations and extra work incident to regular plant operation.

It is only natural that as some cities had much better facilities for conducting experiments than others, the results should reflect those differences, especially in the amount of work accomplished. Nevertheless, all the co-operating cities deserve great credit, and some did painstaking and careful work of excellent quality. Some of them worked only on the first set and some only on the second set of experiments, while others conducted both sets. The following are the cities co-operating: Baltimore, Md., Chicago, Ill., Cincinnati, Ohio, Cleveland, Ohio, Denver, Colo., Kansas City, Mo., Omaha, Nebr., Providence, R. I., Racine, Wis., Richmond, Va., Sacramento, Calif., Springfield, Ill., St. Louis, Mo., Toronto, Ont., Canada, Tulsa, Okla., and Washington, D. C.

Character of Water Used by Various Cities

In order better to understand the conditions prevailing in the cities in which the experiments were conducted, a brief description of the sources from which the water supplies are received, and the methods of treatment, are given herein.

Baltimore, Md.—The water supply of Baltimore is obtained from the Gunpowder River, which has a water-shed above the intake of 303 sq miles. The water is impounded in Loch Raven Reservoir, which holds 23 000 000 000 gal. The water is conveyed through a tunnel seven miles long to Montebello Filters, where it is treated with alum. It then passes through a mixing basin (round-the-end type), and through settling basins of from 3 to 4 hr capacity, and is then filtered. The turbidity of the water coming to the plant during the period covered by the experiments averaged 14 ppm and the water going on to the filters had an average turbidity of 4 ppm. In the latter part of the summer, trouble is generally experienced with the presence of large numbers of micro-organisms, and during the fall months the presence of manganese in the water necessitates a change in treatment from alum to lime and iron.

Chicago, Ill.—The experiments at Chicago were conducted in a large, well-constructed plant, planned for experimental work, and provided with adequate mixing and settling facilities. The raw water was taken from Lake Michigan, at 68th Street. It is generally very low in turbidity, but extremely high in the number of micro-organisms. Alum was used as a coagulant.

Cleveland, Ohio.—The experimental filters at Cleveland were set up at the Division Avenue Filtration Plant, which receives its supply from Lake Erie. The water passes through mixing basins and settling basins before going to the filters. The raw water is usually rather low in turbidity, averaging 19 ppm, but very high in micro-organisms. The turbidity of the water going on to the filters averages about 5 ppm.

Kansas City, Mo.—The water supply of Kansas City is taken from the Missouri River, and varies greatly in turbidity, reaching a maximum of about 20 000 ppm. The water passes through a fully equipped purification plant, and the average turbidity of the water going on to the filters is about 30 ppm. Alum is used as a coagulant. After leaving the filters, the water is pumped through 2 miles of 90-in. pressure tunnel, and several miles of pressure supply lines to large reservoirs, before it is pumped into the distributing system.

As one of the co-operating cities, Kansas City did a large amount of painstaking and careful work, but a study of the results obtained from the experimental filters revealed characteristics different from those obtained in any other place. Dr. George F. Gilkerson, who has charge of the laboratory, states:

"The average raw turbidity at Kansas City plant is approximately 3 000 parts per million. This load is carried by the purification works as follows: presedimentation, 85%; primary coagulation, $12\frac{1}{2}$%, final coagulation, $1\frac{1}{2}$%; filters, 1%.

"The above results would indicate that the average turbidity of the applied water was approximately 30 parts per million. This turbidity is ordinarily caused by a very large percent of very finely divided suspended matter and a relatively small amount of chemical floc. An operator of a plant handling a comparatively clear supply would unhesitatingly state on visiting our plant that the applied water was undercoagulated, and in no way suitable for filtration. The results obtained in our plant, however, do not bear out this assumption.

* * * * * * * * *

"Some conclusions are drawn [from results with experimental filters in other cities], which, from the data collected, are unquestionably true. The writer, however, feels just as positive that the results obtained [in these experiments] could not possibly be obtained in a plant like the Kansas City, Missouri, plant, without applying a chemical dose to the water that would be inhibitive in cost, and [that would] in no way be compensated for by the increase in filter plant efficiency, or quality of water delivered."

A careful inspection of the laboratory reports of the regular filter plant operation failed to reveal a single instance of any turbidity in the water delivered to the city. The Kansas City filters, due to some particularly favorable features, function properly as a whole, whereas the experimental filters did not. This indicates that the large saving effected at Kansas City by the use of less alum than is generally required, is justified; the only proper criterion for judging plant performance is the quality of the effluent.

Richmond, Va.—The City of Richmond receives its water from the James River, which varies considerably in turbidity, at times reaching 2 000 ppm. The water passes through small mixing basins into settling basins, in which flocculators have been added. From the settling basins it goes to the filters. The turbidity going on to the filters averages about 8 ppm.

Springfield, Ill.—The raw water for the Springfield supply consists of a combination of water from Sangamon River and well water, averaging about 75% river water. It is treated with alum and lime, and occasionally with

activated carbon. It passes through circular mixing tanks with a normal detention time of about 40 min, and thence through clarifiers to settling basins having a total capacity of about 6 hr. The water is recarbonated to a pH of about 9 before going to the filters.

St. Louis, Mo.—The experimental filters at St. Louis were set up at the Chain of Rocks Filters, which is supplied from the Mississippi River. The turbidity of the raw water varies greatly, averaging about 1 400 ppm. It passes through long mixing chambers into large settling basins before going to the filters. The average turbidity of the water going to the filters is 9 ppm. Lime, iron, and alum are used as coagulants.

Toronto, Ont., Canada.—The water supply of Toronto is obtained from Lake Ontario, and has an average turbidity of about 6 ppm. At certain times, the water runs fairly high in micro-organisms. For experimental work, a specially designed plant was constructed, which receives its water from the raw-water pipes feeding water to the Island Filter Plant. The water first passes through a mixing trough, where alum is added. From the trough the water circulates through spiral mixing chambers before passing to the settling basins. A flume then takes the treated water to the filters.

Tulsa, Okla.—The water for Tulsa is secured from Spavinaw Creek and impounded in Mohawk Lake, holding about 500 000 000 gal. It is conveyed thence for 55 miles, through a concrete conduit from 54 to 60 in. in diameter. At the plant it passes through two circular basins, is aerated, and then passes through two coagulating basins having a detention time of about 3 hr. Alum is used as a coagulant and lime is used occasionally before the water goes to the filters. The turbidity of the raw water is ordinarily about 15, but it occasionally reaches about 1 000, ppm. The turbidity of the settled water is from 2 to 15 ppm.

Washington, D. C.—Water for the City of Washington is taken from the Potomac River above Great Falls, and is conveyed thence through several miles of gravity conduit to Dalecarlia Lake, where some of the silt settles out. The turbidity of the raw water varies from 6 to 2 000 ppm, and sometimes changes suddenly. The water passes through a mixing chamber, settling basins, and a fully equipped filter plant. An average of 1.5 grains of alum per gal is applied as a coagulant.

Description of Apparatus

The apparatus used by the various cities in conducting the experiments was substantially the same, but varied somewhat in minor details. It consisted of a battery of eight small glass filters made of pyrex glass tubing, approximately $1\frac{3}{4}$ in. in inside diameter, and from 5 to 6 ft in length (see Fig. 12). The top and the bottom of each tube were closed with rubber stoppers. Water was admitted through a small glass tube passing through the upper stopper; it was withdrawn from the bottom in a similar manner. The influent tubes were extended above the maximum water level to vent the filters.

Each of the units was equipped with a mercury manometer, for determining loss of head, and a filter rate controller. A thermometer was fixed in the

wash-water supply line. Brass needle-valves were used to control the application of the wash-water, and glass fittings, connected with rubber tubing, served for most of the pipe connections.

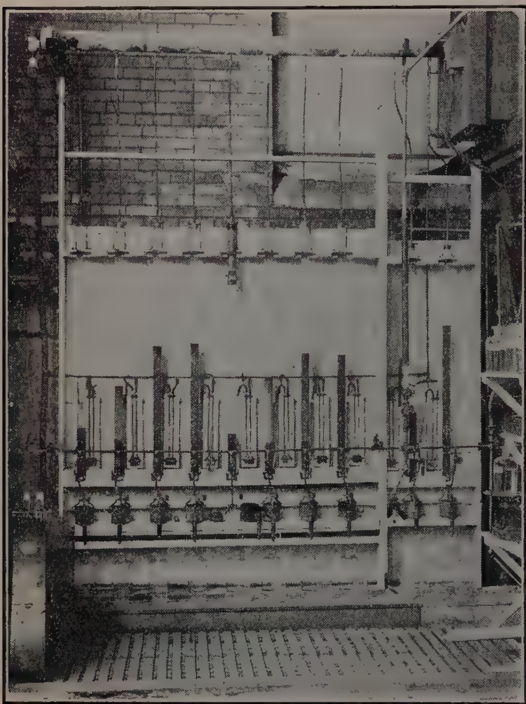

FIG. 12.—EXPERIMENTAL FILTERS USED AT TORONTO, ONT., CANADA.

This experimental assembly was generally placed in some position in the plant where the water level on the test filters would be identical with that in the plant units. The wash-water was supplied from one of the pressure mains and was regulated by two control valves. Different cities worked out special details to meet particular needs. For instance, at Tulsa, where the filters were in a rather inaccessible place, a bell-ringing device was installed to notify the filter attendant when a loss of head of 8 ft had been reached. At Toronto, where conditions permitted its use, a "potato masher" for breaking up sand for the secondary wash was permanently installed.

The filter rate controller used by most of the co-operating cities (Fig. 13) was devised by Mr. George C. Dobler, of the Montebello Filters, at Baltimore. This controller proved satisfactory and accurate and was not difficult to make. The orifice was first calibrated to give approximately the desired rate, the more exact rate having been determined by shifting the orifice above or below the constant level in the beaker until the desired rate was secured. At Richmond, Chicago, and Toronto, controllers of individual local design were used.

Determination of Sand Sizes.—After the sand for use in the experimental filters had been graded by mechanical shaking, the size of grain passing each sieve was determined as follows: On each sieve was placed a small sample of the grade that had previously passed the same sieve, mechanically shaken. This time the sieve was shaken by hand; the last few grains passing were collected, and 100 of them were measured under a microscope. The measurement taken was of the narrowest side exposed to view. The average of the 100 grains was taken as the "size passing," or "size of separation." The size of

any sand grade was considered as the average of the size of separation of the sieve through which it all passes and that of the sieve upon which it is all retained.

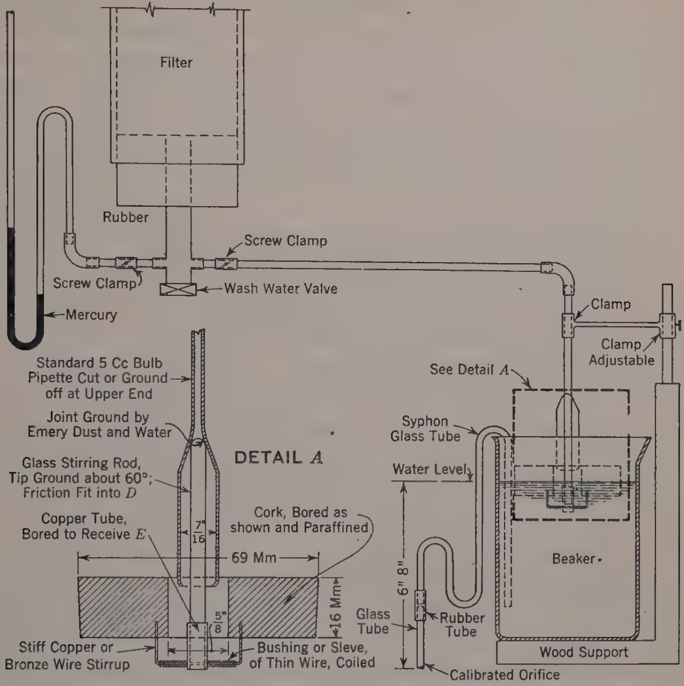

FIG. 13.—RATE CONTROLLER FOR EXPERIMENTAL FILTERS.

Calibration of Glass Filter Tubes.—It was found that the filter tubes were not all of exact internal diameter for the full height, and that the diameter of the different tubes varied as much as 0.27 sq in. in area. Consequently, it was necessary to calibrate each filter tube before it was put into service. Particular attention was paid to the determination of the exact area at the level of the top of the sand layer, in order that the surface of the filter medium could be known accurately. On account of the practical impossibility of making direct inside measurements, the following method was used: Marks were made on the glass at points between which the average areas were to be determined. The tube was then filled with water, and the water between any two of these marks was carefully withdrawn and measured in a graduate. The average area was then determined by dividing the volume of water withdrawn by the depth it occupied in the tube.

Placing Sand in Filter Tubes.—In the earlier experiments, much trouble was experienced in establishing a zero mark for the sand-rise curves in the different filter units. It was found to be due principally to the manner in which the sand was originally placed in the filter. The following procedure eliminated the difficulty: Before placing sand in the tubes, a mark was made

on the glass at the exact height to which each layer was to be brought. With the wash-water valve slightly open, a quantity of sand was added; that valve was then closed, and the effluent valve was suddenly opened wide. If the sand did not settle exactly to the mark more was added or some was removed, and the process was repeated. This operation was performed for each layer, and the last layer established the correct zero mark for subsequent operations.

Gravel.—It was found by experiment that gravel, other than very small sizes, was not easily displaced in the small glass tubes. A layer, 6 in. deep, properly graded to support the sand, was sufficient to resist the highest velocity of wash-water.

Technique of Experiments on Sand of Uniform Size

For the experiments on sand of uniform size, the authorities of Baltimore used the same eight grades that had been used in the previous set of experiments. Only one grade was placed in each filter tube. The size of sand reported was the average of the size passing and the size retained on a given sieve. At Toronto, the sand was re-screened through Tyler square root sieves. Six grades were used, and the size reported was that given as the rating of the sieves and was considered as the size passing. However, in plotting curves, to be compared with those from Baltimore, the average of the size passing and the size retained was used.

On account of the necessity of frequently removing and replacing the sand and the desirability of having a definite starting point for beginning each run, the use of gravel as a supporting medium was abandoned. At Toronto, the sand was supported on a layer of fine shot. Baltimore used a wire screen, soldered to the top of a short section of brass pipe that fits inside the tube and could easily be adjusted to the correct position. The pipe was turned on a lathe to approximately the inside diameter of the tube, and was slit longitudinally to make it slightly flexible.

Silt Penetration.—The distribution of silt deposited in a filter bed in a single run, and the depth of its penetration, are determined, as follows: Upon completion of a run, all the sand is removed in small sections by means of a siphon, and each section is deposited in a separate beaker. In the fine-grained filters, the sections are 0.5 in. in depth, but as the sand size increases, the depth of sections is increased to 1 in. and 2 in. About 300 cu cm of filtered water is used in siphoning a 0.5-in. section of sand from the tube, and 500 cu cm and 600 cu cm, respectively, are used for 1-in. and 2-in. sections. The sand and water are caught in a beaker and stirred thoroughly with a glass rod. The water is then poured into a graduate, care being taken not to remove any of the sand. About 200 cu cm of filtered water is then added to the sand. The stirring is repeated, and the washing is poured into the graduate. These operations are continued until the total washings measure 1 000 cu cm. All the water is then transferred to a bottle, and thoroughly shaken, and the turbidity is immediately read in a Jackson turbidimeter. Turbidity readings for washings from 0.5-in. sections are recorded as read. Readings for the 1-in. sections are divided by two (the equivalent of

diluting to 2 liters), and the 2-in. sections are divided by four (the equivalent of diluting to 4 liters). The reasons for doing this are that in the 1-in. and 2-in. sections, the turbidity is more easily read without further dilution, and it is difficult to break down the floc properly if the large volume of water required for dilution of the deeper sections is used.

It is necessary to make corrections in the turbidity readings to compensate for accidental variations in depth of the sand sections removed by siphon. Hence, after each section has been washed carefully, it is drained of as much water as possible, thoroughly dried on a hot plate, and then weighed. The ratio of this weight to that of the theoretical section is the required correction factor.

The maximum depth of penetration of floc into the sand bed was defined as that depth beyond which the turbidity of water washed from the portion considered is less than 10 ppm, when the volume of wash-water used is in the proportion of 1 liter to each 0.5 in. of sand depth. This definition was based on the observation that an amount of floc sufficient to produce a turbidity of only 10 ppm under the specified conditions, seems to have no appreciable effect on the filter effluent. The difficulty in washing the sand perfectly clear, of thoroughly breaking up the floc, and of making accurate readings with a Jackson turbidimeter, all point to the conclusion that turbidities of less than 10 should be neglected in determining the depth of penetration.

Precision Siphon.—At Toronto, the laboratory work was simplified by the use of a precision siphon, which could be adjusted to remove the sand between any desired limits.

As the siphon nozzle turns, its cutting-edge is fed spirally downward into the bed. The sand is immediately picked up in the stream of water entering the mouth. On account of the shape of the nozzle, the velocity of the water is low at the cutting-edge and its direction is nearly horizontal, so that it gathers little sand and sediment from in front of the edge. A tape secured to the filter tube indicates that the siphon is quite accurate. Any depth of cut $\frac{1}{4}$ in. or more, can be made accurately and quickly by using the correct gage between the stops.

Secondary Experiments

Sand Rise and Sand Expansion.—Sand rise and sand expansion are terms used to express the vertical rise of sand during the period of washing. Sand rise is usually expressed as rise in inches, while sand expansion is generally expressed as a percentage. With a fixed rate of wash, the sand rise will vary with the depth of the sand bed, the size and grading of the sand, and the temperature of the water. It is evident that the sand rise as determined for any given filter will not apply to another filter unless all the factors are identical.

In a graded sand filter, each grade will expand by a different percentage for each rate of wash-water. At some rates, the coarser grades may not expand at all. In a filter containing sand of one size, the percentage expan-

sion will be a function of the velocity and temperature of the applied wash-water, but with a graded sand the expression can have no definite general meaning, as every change in grading would also modify to some extent the expansion.

Although the term, "sand rise," is open to the same objections, it was used rather than "sand expansion," as it seemed to simplify the recording of results. As sand rise varies with the water rise, the observed data could be plotted directly without making any calculations. Moreover, if the sand rise is considered in relation to the space between the normal sand surface and the edge of the wash-water gutter, the direct reading, in inches, will show at a glance whether there is any possible danger of the loss of sand.

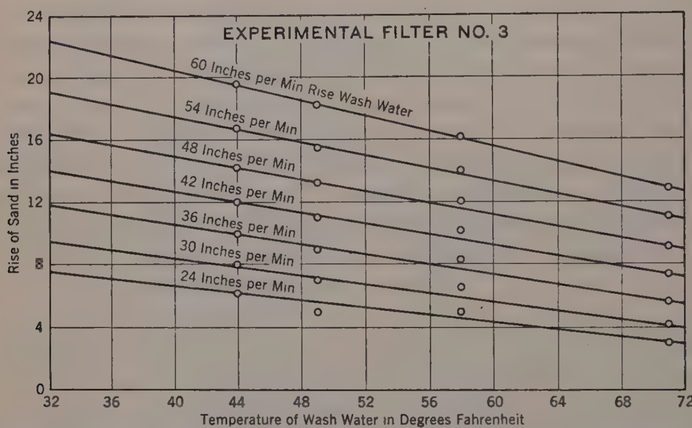

FIG. 14.—RELATIONS BETWEEN SAND RISE, WATER RISE, AND TEMPERATURES OF WASH WATER. (EXPERIMENTAL FILTER NO. 3, SPRINGFIELD, ILL.)

Constant Sand Rise.—Experiments were made at Toronto and Springfield to determine the vertical rise of water, in inches per minute, necessary to maintain a constant sand rise at various temperatures of water. Typical results, plotted in three different ways, are shown in Figs. 14 and 15.

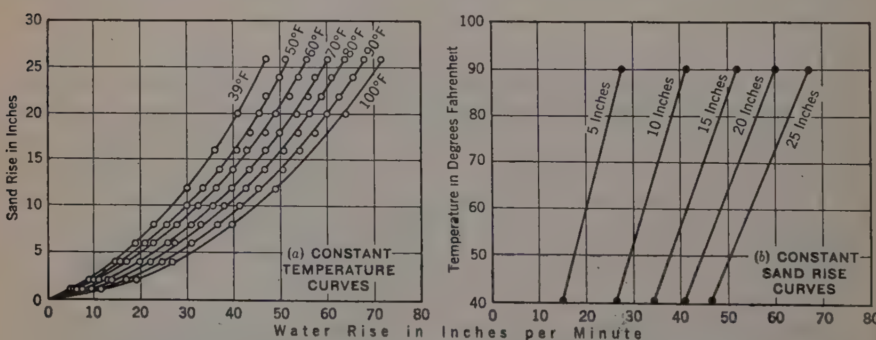

FIG. 15.—RELATIONS BETWEEN SAND RISE, WATER RISE, AND TEMPERATURE OF WASH WATER (EXPERIMENTAL FILTER AT TORONTO, ONT., CANADA).

The results suggest the desirability of changing the wash rate to correspond with the temperature changes. The following statements are excerpts from an account of the experiments at Toronto by Mr. Allan:

"Since for every temperature there is a definite value of viscosity, temperature may be used as a measure of viscosity. The temperature-water rise curves may be considered to be the resultant of two curves; namely, the viscosity-water rise curve and the temperature-viscosity curve. These two curves being concave in opposite directions, produce a resultant which is probably a straight line or nearly so.

"When water rise is plotted against temperature, with sand rise constant, the points appear to be in a straight line. If this can be proved to be true, or if it can be shown that the points are not far from a straight line, for all sand sizes and gradings, a very useful formula will result. These straight line graphs possess all the information necessary to plot sand rise curves, and if the straight line relationship holds, then it would only be necessary to take two readings, and join the plotted points with a straight line produced.

"The practical use to which these curves could be put would include the following. Suppose that under plant conditions washing is carried out using a constant sand rise at all times, and it is desired to know the water rise necessary to produce the specified sand rise at the known temperature of the water. This information is readily available by consulting the 'straight line' graphs."

Hydraulic Grading.—It is generally assumed that the hydraulic grading of the sand in a rapid filter, after washing, is perfect. This, however, is only approximately true. The closeness of hydraulic grading depends upon several factors, the most potent one being the rate at which the wash-water valve is closed. When a filter is being washed, especially at high rates, there is a general mix-up in the sizes of sand. If the wash-water valve is closed suddenly, the heavier particles carry down with them some of the finer particles, but if the valve is closed gradually, there is generally sufficient velocity to maintain the smaller particles in suspension, while the coarser ones settle into position. The size of the sand grains also has something to do with the hydraulic grading. It has been noted in filters containing fine sand, that the grading is much more perfect than in those containing a well graded mixture of fine and coarse sand.

The action of sand under the influence of wash-water is as observed in the small glass filters. The jets, or currents, of water leaving the gravel surface are not uniform in direction or intensity. The sand grains when acted upon by a jet are lifted to different heights, depending on their size and shape. When a grain reaches the highest limit to which a jet can lift it, it is shifted to one side into a current of lesser velocity, and settles down to a point where it is again picked up. In all these movements there is conflict with other grains, tending to lift them higher or to force them lower into the bed. The fine grains that have been lifted near the top are no longer subject to the same intensity of jet action, because the greater sand expansion has increased the water area and the rising currents have been reduced to a more uniform velocity. The wide space between sand grains gives plenty of room for movement without interference to the neighboring grains, and, consequently, each grain can more nearly take its proper place in the hydraulic scale.

In addition to the visual inspection of the filter beds during the ordinary washing period, colored sand was placed in some of the filters, and, in others, sections of the sand were removed and analyzed. All three methods confirmed the fact that the sand was more or less imperfectly graded.

Viscosity.—The viscosity of water plays a far more important part in the operation of filters than has generally been supposed. There is a critical range in the temperature of water from 33°F to about 45°F when its viscosity increases about 18 per cent. Within this range floc forms less compactly, and settles out more slowly, than at higher temperatures. Unless the coagulant is applied in exactly the correct quantity, the water going on to the filters will be treated improperly and the floc will pass through the sand bed.

When the water is cold and more viscous, the filter runs are longer and the sand is lifted higher with the same velocity of wash-water, making it easier to keep the filter in good condition. At low temperatures, a drop in temperature of only 5 or 6° will cause an increase of several inches in the sand rise. Water has its maximum lifting power near the freezing point, where its viscosity is the greatest.

Ratio of Critical Depth to Sand Size.—The experiments seem to indicate that the critical depth of a sand varies as the square of the diameter of the grains (see Table 2). The Baltimore results conform more nearly to this relationship than those at Toronto, partly, perhaps, because of the greater number of runs. In Table 2, the average depths of penetration were used instead of the maximum, since any general law, where as many variables are encountered as in filter experiments, can only apply to average conditions.

Turbidity Determinations.—All turbidities were determined optically and expressed in the usual way, in parts per million, although it is fully realized that the expression, "parts per million", has no real meaning in

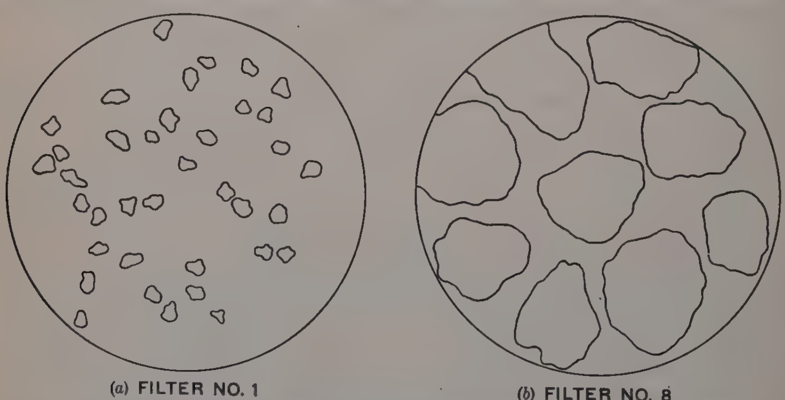

FIG. 16.—SHAPE OF SAND GRAINS USED IN EXPERIMENTAL FILTERS. (MAGNIFICATION, 5.6 DIAMETERS).

such cases. In order to learn any possible relationship between turbidities determined optically and those determined by evaporation and weighing, the records made at Montebello Filters, during four years of regular plant operation, were studied. A glance at Fig. 16 will show that there is no

relationship between turbidity as determined optically and by weight. The laboratory method used was to determine optically the turbidity of a 500-cu cm sample of water, and then to filter the sample through a previously weighed crucible packed with asbestos. Next, the crucible was dried at 105° C for 12 hr. The difference in weight of the crucible before filtering and after drying, is the weight of the suspended matter in the water. Fig. 16 contains information involving more than 1 400 separate determinations, each dot representing the monthly average of daily tests. As defined, the coefficient of fineness is the weight of suspended matter, in parts per million, divided by the optical turbidity, "in parts per million."

Turbidimilligram and Accumulated Turbidity.—"Turbidimilligram" is the term used at Toronto to express the turbidity contained in water when the absolute weight is unknown. As defined by Mr. Allan, it is the weight of the suspended matter in 1 liter of water having a turbidity of 1 ppm. The weight of turbidity in any body of water is computed by multiplying the volume, in liters, of the water by its turbidity, in parts per million, the result being expressed as turbidimilligrams.

The term, "accumulated turbidity," used at Baltimore in plotting the penetration curves, means the summation of the turbidity readings of the 1 000 cu cm washed from all sections above the point in question While the term of itself has no particular meaning, it does have a very definite meaning in plotting the curves.

Data on Sand Used in Experiments.—The following data were secured at Baltimore for studies made in connection with the experiments on uniform size sand: A small sample of each grade was selected and thoroughly re-screened. Fifty grains from each of the eight different sizes were then selected at random and weighed carefully, and the average weight was com-

TABLE 7.—Essential Data on Sand Used in Experimental Filters
(All Averages Are for Fifty Sand Grains)

Description	Filter No.							
	1	2	3	4	5	6	7	8
Total weight of fifty sand grains, in milligrams	5.00	7.90	12.00	14.30	17.00	41.80	84.20	565.30
Average weight of one sand grain, in milligrams	0.10	0.158	0.24	0.286	0.340	0.836	1.684	11.306
Average width of one sand grain, in millimeters	0.34	0.41	0.48	0.61	0.70	0.93	1.13	2.10
Maximum width of one sand grain, in millimeters	0.50	0.54	0.66	0.75	0.91	1.12	1.37	2.66
Minimum width of one sand grain, in millimeters	0.25	0.25	0.37	0.50	0.58	0.66	0.83	1.74
Average length of one sand grain, in millimeters	0.47	0.54	0.66	0.84	0.91	1.27	1.51	2.81
Maximum length of one sand grain, in millimeters	0.66	0.83	1.08	1.25	1.37	1.66	1.91	4.15
Minimum length of one sand grain, in millimeters	0.33	0.37	0.46	0.58	0.66	0.87	1.08	2.20
Average time of settling through 36 in. of clean water, in seconds	24.70	21.20	17.60	14.20	11.70	9.30	7.70	4.90
Maximum time of settling through 36 in. of clean water, in seconds	32.60	26.00	25.00	18.00	15.80	12.40	12.40	7.40
Minimum time of settling through 36 in. of clean water, in seconds	20.20	17.00	13.00	12.00	9.80	7.80	5.20	3.00
Temperature of water, in degrees Fahrenheit	78.00	78.00	78.00	76.00	76.00	78.00	78.00	78.00

puted. The longest and narrowest dimension of each grain was then carefully measured under a microscope. These weights and measurements are summarized in Table 7. Considering the average minimum width of the grains in each group as the size of the sand, the sizes thus determined checked closely with those reported as placed in the experimental filters, except for Filter No. 8. In this case the measured size was somewhat larger, probably because of the small number of grains selected, but it was still within the limit of the size passing.

Next, each of the fifty grains for each of the eight sizes was carefully wet to prevent air bubbles from adhering and was then dropped into a small glass tube filled with water, upon which marks were made exactly 36 in. apart. The time that it took each of these grains to traverse the distance between the marks was obtained by a stop-watch and the average was computed (see Table 7). Especially in the larger sizes, some of the grains did not settle in a straight line, but zigzagged back and forth across the tube. This movement, evidently produced by their unusual shape, accounts for the long time it took some of the grains to traverse the 36-in. space.

Before these data were secured, a number of grains from each of the eight sizes were placed under a microscope, equipped with a euscope projector. The grains were magnified 12 diameters and the outline of each was traced. Typical results, reduced to a magnification of about 6 diameters are presented in Fig. 16.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

A DIRECT METHOD OF MOMENT DISTRIBUTION

Discussion

BY MESSRS. A. A. EREMIN, AND T. Y. LIN

A. A. EREMIN,[43] ASSOC. M. AM. SOC. C. E. (by letter).[43a]—The method of distributing bending moments in rigid frames described by Mr. Lin has considerable merit. It gives direct distribution of bending stresses from the moment at one joint. Likewise, the method is convenient for the construction of influence lines for bending stresses in rigid frames.

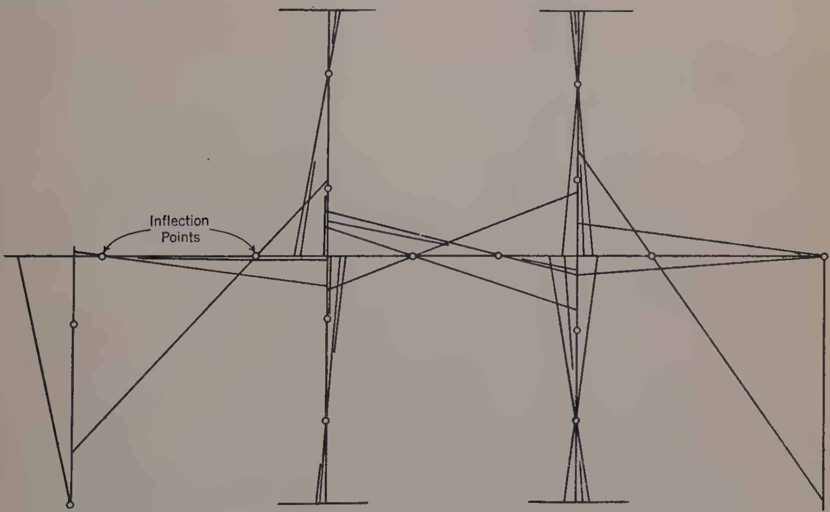


FIG. 25.—DISTRIBUTION OF BENDING MOMENTS.

The computation of bending moment stresses in a frame with two or more spans loaded may be simplified by applying graphical methods. For

NOTE.—The paper by T. Y. Lin, Jun. Am. Soc. C. E., was published in December 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1935, by Messrs. L. E. Grinter, W. H. Huang, Felix H. Spitzer, Harold E. Wessman, Egor P. Popoff, and L. T. Evans; May, 1935, by Messrs. C. S. Salter, Leon Blog, Austin H. Reeves, E. J. Bednarski, John T. Howell, and I. Oesterblom; and August, 1935, by W. P. Li, Jun. Am. Soc. C. E.

[43] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[43a] Received by the Secretary August 20, 1935.

example, consider Example 3 of the paper. In Fig. 25 the lengths of beams and columns are assumed equal to 1. Points of contraflexure were determined with the modified carry-over factors shown by the author in Fig. 11. Bending moments were distributed at joints using the modified stiffness factors shown by the author. Carried-over bending moments were determined graphically by means of the points of contraflexure in Fig. 25. The resulting end bending moments in the members will be equal to the algebraic sum of moments scaled from Fig. 25 and fixed-end moments.

Conventional rules for signs of moments in Fig. 25 are similar to those introduced in the paper. Positive bending moments at joints are considered clockwise and negative moments, counter-clockwise. The use of Fig. 25 helps to avoid an error in the signs of bending moments. By graphical methods, the distribution of moments can be visualized readily. Furthermore, graphical construction reduces the work of distributing moments.

T. Y. Lin.[44] Jun. Am. Soc. C. E. (by letter).[44a]—Judging from the interesting discussions of this paper, many engineers have evidently found it valuable. Professor Grinter makes a thoughtful comparison with the original method; but he does not compare the two on the same basis. The members in Fig. 14 are all of uniform moment of inertia, whereas in Figs. 8 to 11, some are not. To make a fair comparison, the writer will solve Example 3 by a simplified Cross method which, he believes, requires the least amount of written work possible with that method. Instead of writing both the distributed and carried-over moments at the same time, as Professor Grinter does, only the carried-over moments are written during the process of releasing joints (see Fig. 26(*b*)); the distributed moments are to be computed and written at the end of the procedure as suggested by Professors Cross and Morgan.[45] The writer proposes to give values of the product, $K\,\gamma$ (Fig. 26(*a*)), in addition to the stiffness ratio, K (which is the proportion of distribution of each member at a joint; that is, $\dfrac{K \text{ of end of member}}{\Sigma K \text{ of ends of all members at joint}}$), so that when releasing a joint one simply multiplies the unbalanced moments at that joint by the value of $K\,\gamma$ for that end of each member and writes it at the farther end. It also facilitates the work to put a check mark on an unbalanced moment (whether a fixed-end or carried-over moment) after it has been released and carried over to the other ends. This will eliminate any confusion in the process of releasing a joint. To release the most unbalanced joint first and to record the sequence of releasing is also very helpful.

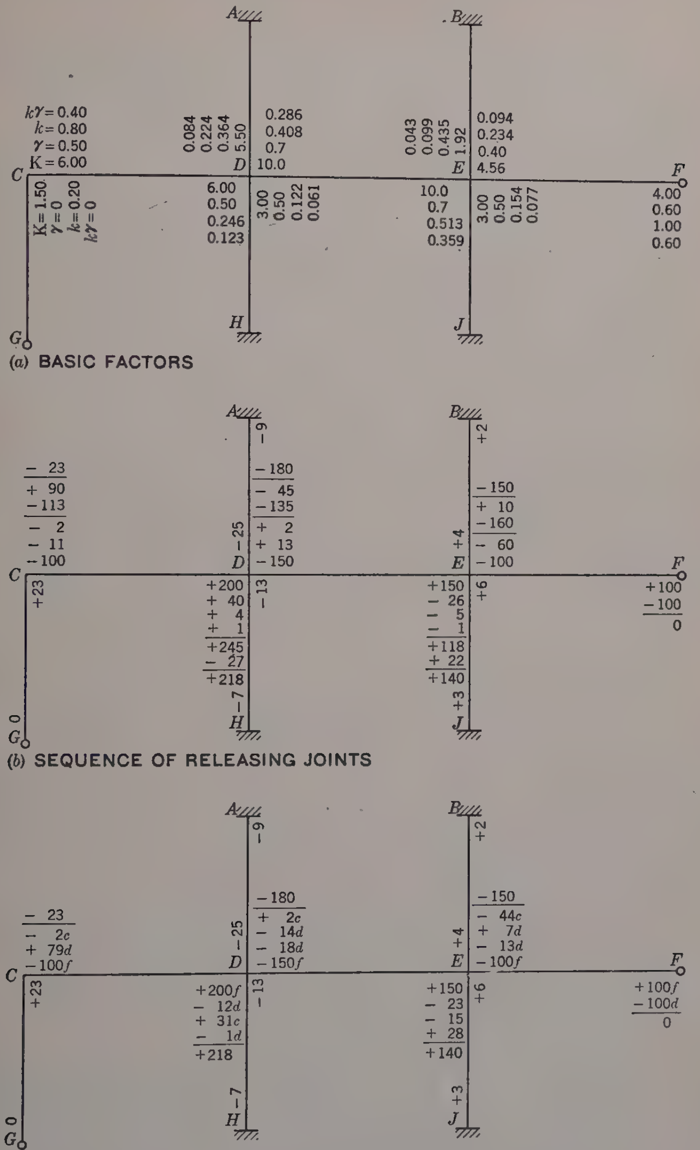
This also requires a certain amount of preliminary work, however. It will be necessary first to record the K and $K\,\gamma$-values for each end which should be released. To simplify the work further, find the modified values of K_{cg}, K_{ef}, K_{da}, K_{eb}, γ_{da}, and γ_{eb} by Equations (3) and (4). Then, for the unbalanced moments, one begins by releasing Joint C, multiplying the -100 (reversed

[44] With Ministry of Railways Chinese National Govt., Care, Cheng-yii Ry., Chungking, Szechuan, China.

[44a] Received by the Secretary October 26, 1936.

[45] "Continuous Frames of Reinforced Concrete", by Hardy Cross and C. E. Morgan, Members, Am. Soc. C. E., p. 104.

in sign) by the value of $K\gamma$ for CD and writing $+40$ at End D. Then, at Joint D, the unbalanced moment $= -150 + 200 + 40 = +90$, which is multiplied by the corresponding $K\gamma$-values and written at the farther ends



(a) BASIC FACTORS

(b) SEQUENCE OF RELEASING JOINTS

(c) SOLUTION BY DIRECT METHOD

FIG. 26.—SIMPLIFIED SOLUTION OF EXAMPLE 3.

of members concerned. For the terminal joints, A, B, F, G, H, and J, the moments will be carried over at the end of the procedure. Continuing in

the sequence shown, it is found that the values are converging rapidly, giving close results at the end of the ninth cycle. The unbalanced moment at each joint is then added and distributed to the ends of members meeting at that joint in proportion to the K-values. Thus, at Joint D, the unbalanced moment is $245 - 135 = 110$, which gives -25 to Column DA, -45 to Girder DE, etc. The final moments are then as shown in Fig. 26(c). Moments at Point A of Column AD and Point H of Column HD can now be found to be -7 and -9, respectively.

To do justice to the direct method, it should be noted that Fig. 11 has been made voluminous purposely in order to give the reader an unmistakable understanding of the procedure. It can easily be simplified if one carries the calculation to three significant figures and obtains moments for DA, DH, EB, and EJ by a single distribution at the end of the procedure. For example, at Joint D, $218 - 180 = 38$; $-38 \times \dfrac{5.5}{5.5 + 3.0} = -25$ for DA; and $-38 \times \dfrac{3.0}{5.5 + 3.0} = -13$ for DH. Then the computation will appear as in Fig. 26(c).

By comparing Fig. 26(b) and Fig. 26(c), there is seen to be little to choose between the two procedures. Each is simplified as much as possible. Perhaps the former requires a little more balancing, but less preliminary work. It must be noted that in Fig. 26(b), equations of the direct method have been applied to four members. Had this not been done, more calculation would appear.

The foregoing is a comparison of a typical case, but by a more extensive study, one can draw the following conclusions regarding the simplicity of the direct, as compared to the original, moment-distribution method:

(1) In many cases, especially where several conditions of loading are considered or where some members are of varying moment of inertia, this direct modification is simpler than the original method.

(2) When the original method is simplified to the utmost extent, it compares favorably with this direct method in most cases.

(3) Either method has special applications to which it is best adapted, and, in many cases, there is not much to choose between them; but the simplest method is generally a combination of both, if one is thoroughly acquainted with their fundamental characteristics.

Professor Grinter's experience with the use of "end rotation constant" is probably correct; but it is no evidence against this much more general direct method. Mr. Howell has indicated clearly the difference between the two.

Curves, nomographs, or tables were used by the writer in the solution of Equations (3) and (4). Curves such as those in Fig. 27 provide a better means of visualizing the different factors. Stiffnesses, carry-over factors, and fixed-end moments of haunched beams can be found in many books and pamphlets.

With reference to side-sway, the direct method can be applied in the side-sway problem in the same manner as was done in the original Cross method. It is not necessary to describe the application.

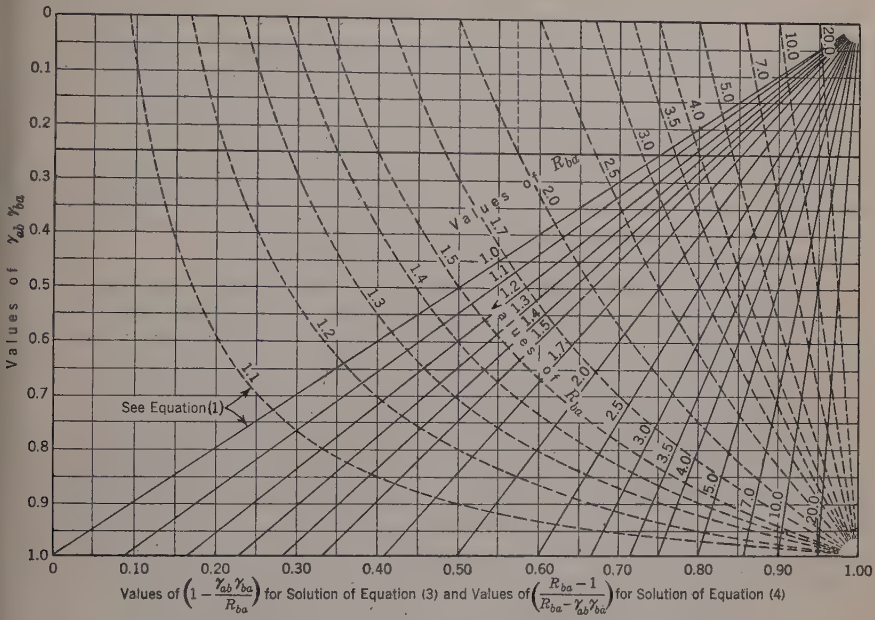


FIG. 27.

Professor Grinter and Mr. Huang both suggested the idea of using $\frac{K_{ba}+\Sigma K_{bn}}{K_{ba}}$ instead of $\frac{K_{ba}+\Sigma K_{bnm}}{K_{ba}}$. Mr. Huang has further found the resulting error in the value of K_{abm} to be less than 2.08 per cent. However, the writer finds that for haunched members the $\gamma_{ab}\,\gamma_{ba}$-value of which is greater than 0.25, the resulting error can be large, perhaps even more than 25 per cent. Hence, the writer prefers the use of estimated values of K_{bnm}. With only a little practice, K_{bnm}-values can be estimated quite closely and the value of K_{abm} can be obtained from graphs with practically no error.

The writer regrets that Mr. Evans' discussion, interesting as it is, is apparently based on a misunderstanding. Equation (1) is simply a definition of the term, R_{ba}, introduced to simplify Equations (3) and (4). It cannot be incorrect, as Mr. Evans claims, although it could appear in some other forms with corresponding changes in Equations (3) and (4) as Mr. Popoff states. That modified stiffness should be used for Members B_1, B_2, and B_3, but not for Member BA, is evident in Step 2 of the paper in which the writer took special care to point out that when Joint B is released, only Joint A is fixed, not the others. Hence, it is entirely wrong to apply the modified stiffness for Member BA as Mr. Evans has done in the major part of his discussion.

The product, $\gamma_{ab}\,\gamma_{ba}$, refers to the carry-over factors of two ends of a member. It only changes with the moment of inertia variation, but has nothing to do with the degree of restraint at the ends of the member; R_{ba} is a term denoting the relation between K_{ba} and ΣK_{bnm}. As long as K_{ba} remains unchanged, it has nothing to do with the moment of inertia variation.

Many discussions have appeared regarding similarity of methods and priority of concept. Although some are correct to a certain extent, others have evidently been advanced without a sufficient study of the paper. Certainly, the more one knows, the more one comes to believe that there is "nothing new under the sun." There are many methods similar to that proposed in this paper; however, none seems to be as widely applicable and as simple as this direct method, and none is derived from the principle of moment distribution.

The conjugate point method[29] is much like the German method—"Die Methode der Festpunkte"[46]; but all who have compared it with the moment distribution method agree that as a tool of analysis the latter is by far the better. This means that it cannot compare with this direct method, which can be much more easily learned and more widely applied.

It is to be regretted that the writer cannot obtain Dr. Zimmermann's book, mentioned by Mr. Oesterblom, and the thesis presented by Mr. A. Efsen to the University of Copenhagen in 1930 in fulfillment of the requirements for the degree of Doctor Technices, to which Theodore B. Host, Assoc. M. Am. Soc. C. E., has called the writer's attention by correspondence; but he infers from the opinions of the discussers that the methods are not derived in the same manner and probably cannot be as easily and generally applied.

Mr. Huang[12] has a paper published in Chinese that advances a similar method derived from slope-deflection equations. It is similar to other methods derived therefrom, but is more cumbersome. Probably the method presented by Mr. E. B. Russell[47] is most similar to that proposed by the writer. This method is developed from slope-deflection equations and introduces two rather complicated equations for finding the end moments. The same is true of the method[48] of T. F. Hickerson, M. Am. Soc. C. E., and that[49] proposed by H. M. Hadley, Assoc. M. Am. Soc. C. E.

In the original thesis,[8] there is a brief, but rather extensive, comparison of all methods of continuous frame analysis. It might prove interesting to one who is scholarly enough to make further investigations. After long study of, and experience with, the subject, the writer has come to the conclusion that although each particular problem may be solved best by a certain particular method, it suffices for all purposes to have learned the Cross method and this direct method. The latter is most valuable in the visualization and study

[29] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

[46] "Berechnung der Statisch-unbestimmte Systeme", von A. Strassner, Berlin, 1921.

[12] *Journal*, The Chinese Inst. of Engrs., Vol. 9, No. 5.

[47] "Analysis of Continuous Frames", by Earle B. Russell, San Francisco, Calif., 1934.

[48] "Structural Frameworks", by T. F. Hickerson, Univ. of North Carolina Press, 1934.

[49] "The Assembly Stiffness Method of Stress Analysis", 8 pages of notes by H. M. Hadley.

[8] Thesis by T. Y. Lin presented to the Univ. of California in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering, March, 1933.

of elastic properties of continuous frames and in the approximate but sufficiently accurate estimate of moments in them. Any one who has learned the fundamentals of this method should be able to "write in" moments to a sufficient degree of accuracy. He should be able to see mistakes in analysis. He should be able to visualize the effect of haunching; the effect of restraint; and to see a continuous frame as he sees a simple truss. He should know what approximation can be applied in analysis, and where changes in design can be made for economy.

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DISCUSSIONS

LATERAL PILE-LOADING TESTS

Discussion

By Lawrence B. Feagin, Assoc. M. Am. Soc. C. E.

Lawrence B. Feagin,[35] Assoc. M. Am. Soc. C. E. (by letter).[35a]—A wide range of thought and of methods of considering the problem of the reaction of piles to lateral loads is reflected in the discussions of the paper. The interest and consideration accorded it are gratifying and appreciated. The problem has been approached both from the standpoint of mathematical analysis and from that of experience.

Mr. Cummings and Mr. Chang have both contributed ingenious mathematical analyses. They have sought to derive equations whereby the following may be determined: (*a*) The curve of deflection of the central axis of a pile subjected to a lateral load; (*b*) the horizontal deflection of a pile subjected to a given lateral load; (*c*) the length of the pile from the top downward, which is moved, also referred to as the bent length; (*d*) the relationship of the portion of the load resisted by the rigidity of the pile itself to that resisted by the soil; and (*e*) the stresses in the pile.

The writer has studied, with interest, each of these discussions as well as the assumptions made in each, and believes that perhaps a brief comparison may be of assistance to those who may hereafter have occasion to refer to them.

Mr. Cummings has assumed that the equation of the central line of the deflected pile can be expressed by the power series, Equation (1), and assumes certain boundary conditions whereby the formula of the deflection curve of the bent pile is established in the form of his Equation (2).

Mr. Chang has applied the theory presented by Professor S. Timoshenko[36] for determining the deflection of a bar supported along its entire length by a continuous elastic foundation, and has arrived at a corresponding formula of the deflection curve of the pile (his Equation (36)).

Note.—The paper by Lawrence B. Feagin, Assoc. M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by A. E. Cummings, Assoc. M. Am. Soc. C. E.; January, 1936, by Messrs. J. C. Meem, and T. Kennard Thomson; February, 1936, by August F. Niederhoff, Jun. Am. Soc. C. E.; August, 1936, by Messrs. Lazarus White and Y. L. Chang; and September, 1936, by D. P. Krynine, M. Am. Soc. C. E.

[35] Senior Engr., U. S. Engr. Office, St. Louis, Mo.

[35a] Received by the Secretary November 12, 1936.

[36] "Strength of Materials", by S. Timoshenko, Pt. II, p. 402.

It will be noted that Equation (36) is of a form which will produce a sinusoidal curve of diminishing amplitude. In general, this latter type of equation will give a curve which is more nearly in keeping with the actual movement of the pile.

After proceeding with his analysis far enough to determine an expression for the bent length of the pile, Mr. Chang, however, concludes that the pile is practically vertical and rigid at a depth of one wave length, thus indicating that Mr. Cummings was justified in assuming that for all practical purposes the deflection curve has a vertical tangent at depth, L.

In order, for comparison, that it may have the same unknowns as occur in Equation (36), by substituting from Equation (31), Equation (2) becomes:

$$y = \frac{3\, f\, x^2}{\left(\dfrac{216\, E\, I\, \omega}{U}\right)^{\frac{2}{5}}} - \frac{2\, f\, x^3}{\left(\dfrac{216\, E\, I\, \omega}{U}\right)^{\frac{3}{5}}} \quad \text{..............(58)}$$

Equations (36) and (58) both have two unknowns which must be determined by test; that is, f and E_s, the elastic modulus of the soil. Mr. Cummings assumes that the elastic modulus of the sand increases directly with the depth of the sand, whereas Mr. Chang assumes that the elastic modulus is a constant. The writer believes that the former assumption is more nearly correct, notwithstanding the pôssibility, as suggested by Mr. Chang, that the elastic modulus may be affected by the compaction of the soil in the driving of the piles.

Mr. Cummings determined the relationship between the horizontal load, H, and the top deflection of the pile as expressed by his Equation (26). It is interesting to note that in this formula the first term, representing that part of the load carried due to rigidity of the pile, is much less in magnitude for large values of L, than the second term which represents the part carried by the soil. This indicates the important rôle played by the soil in resisting lateral loads. On the other hand, it will be observed that for small values of f, which it seems reasonable to assume will also be accompanied by correspondingly small values of the bent length, L, the first term will represent a large proportion of horizontal load, H. This indicates the importance of the structural rigidity of the top few feet of the pile if the top deflection, f, is to be kept small. It should be borne in mind, however, that the extent to which results from this equation can be relied on depends primarily upon the accuracy with which the bent length, L, and the elastic properties of the soil (represented by ω) are determined.

Mr. Chang has developed a corresponding expression (see his Equation (55)) which may be written as:

$$f = \frac{H}{4\, E\, I \left(\dfrac{E\, s}{4\, E\, I}\right)^{\frac{3}{4}}} = \frac{H}{\sqrt[4]{4\, E\, I}\ (E_s)^{\frac{3}{4}}} \quad \text{............(59)}$$

In Equation (55) he has combined in a single term the resistance due to the rigidity of the pile and to the passive resistance of the soil. The relia-

bility of this equation is also dependent upon the accuracy of determination of E_s which, in turn, is dependent on the bent length of the pile, as well as the elastic properties of the soil.

It will be noted that both Mr. Cummings and Mr. Chang have developed a single value of bent length of pile and each has used his respective single value throughout in computing deflections for the various loads as a basis for comparison with the test data (see Fig. 20 and Fig. 28). This probably accounts in large measure for the fact that the calculated results by both equations show greater deflections for the lower range of loads than the test results. Mr. Cummings uses a computed bent length of 10.2 ft, and Mr. Chang uses $L = 11.35$ ft. Attention is invited to the fact that the writer stated in the paper that at a load of approximately 30 tons per pile and with a top deflection, f, of $1\frac{3}{4}$ in., it was estimated that the bent length of the pile was about 10 ft. For small loads, it is undoubtedly less.

It is believed to be apparent that the depth to which movement occurs in a pile subjected to a lateral force at the top is dependent upon: (a) The physical characteristics of the pile, such as E and I; (b) the elastic modulus of the soil; and (c) the amount of the horizontal force and the resulting lateral movement at the top of the pile.

It is believed, therefore, that any mathematical expression which purports to give the depth of movement should, either directly or indirectly, contain all these factors. The formula developed by Mr. Cummings is Equation (31) and the corresponding formula developed by Mr. Chang is Equation (40). In each of these two expressions it will be noted that terms for both the physical characteristics of the pile and the elastic modulus of the soil are included, but there is no term for either the amount of the horizontal force, H, or the deflection, f. The results from these formulas are largely dependent on the values assumed for the "dimensionless coefficient, ω," and E_s. Based on these formulas the depth of movement would be the same with a deflection, f, of either 0.25 in. or 6 in. at the top, or with a lateral load, H, of 5 tons or 40 tons. It is obvious that this is not correct. The small-scale model tests made by Mr. Cummings (see Figs. 22 and 23) indicate quite clearly that the large rods had a greater depth of movement than the smaller rods. The top deflection of all three rods was about the same (1 in.), but a greater force, H, was required to move the $\frac{1}{4}$-in. rod than the $\frac{3}{32}$-in. rod, thereby causing a greater depth of movement in the more rigid rod.

Mr. Cummings was evidently aware of this and in his discussion states that "it should not be concluded from this that the bent length is independent of the deflection." He then justifies Equation (31) on the grounds that it is a combination of Equations (5) and (12) both of which are "based on the usual theory of elastic structures in which the displacements are required to be very small in comparison with the dimensions of the structure."

The writer does not desire that the foregoing comments on these two excellent discussions be construed as implying that he does not believe that the mathematical analyses are of practical value; they mark an important beginning in a field in which much remains to be done. He does desire, however,

to point out the limitations of certain of the equations, in order that they may be used with proper caution. The equations for deflection check the particular tests results described in the paper quite well and, for loads within the range normally pertinent to design problems, give results which, subject to the additional considerations given subsequently, appear, in general, to be on the side of safety. Furthermore, a careful consideration of the derivations of the bent length provides an approach for the problem of determining the length of pile required to be driven to resist a given lateral load.

Both Mr. Meem and Mr. Thomson have emphasized very properly the fact that the conclusions given in the paper were distinctly limited to the soil conditions and piling arrangements under which the tests were made. The writer recalls reading at one time that Darwin is reputed to have said in effect that "the greatest danger to scientific investigation is generalization based on limited experience." This statement is especially applicable to investigation of problems relating to soil foundations. It is apparent that when movement occurs in the entire foundation, such as that resulting from the movement of the glacial drift described by Mr. Meem, or from quicksand or wet clay in the subsoil stratification in the cases cited by Mr. Thomson, the results of the tests described in the paper are clearly not applicable. Each foundation should be treated as a distinct problem in itself, and the extensiveness of field investigations and office study made dependent upon the importance of the stability of the foundation.

Mr. Niederhoff has ably presented the results of lateral loading tests conducted under the direction of Major Dwight F. Johns, Corps of Engineers, U. S. Army, District Engineer at St. Paul, Minn., in the vicinity of Lock No. 3, near Red Wing, Minn. An ingenious method simulating a condition of fixation of the top of the pile was used. It is rather surprising that in the soft material such as that described by Mr. Niederhoff no greater lateral movement occurred. This may have been due, in part, to the fact that except for a 4-in. space adjacent to the test pile the ground was frozen to a depth of 6 in., thus confining the soil. Furthermore, the low temperature may have greatly retarded or prevented plastic flow of the foundation soil which might otherwise have occurred. With the exception of Tests Nos. 12 and 16 for which notations of 5-min. intervals are given, the length of time that each test load was sustained is not given. If the loads had been sustained for several days it is possible that greater deflections might have occurred.

Mr. Niederhoff has referred briefly to the lateral movement of 9 in. of a portion of the river wall of Lock No. 5A on the Mississippi River, at a time when the lateral force computed by usual methods was only about 3.5 tons per pile. This force was caused by an unbalanced hydrostatic load which, in the writer's opinion, may have produced a partly quickened condition in the foundation sand, giving rise to a much greater actual active load on the foundation piling, especially on the steel sheet-pile cut-off wall, and, at the same time, reducing the passive resistance of the foundation sand. This condition, combined with the presence of slippery clay and the vibration from pile-driving in the immediate vicinity, can easily account for the movement. The presence of wet clay alone might have caused it. Under these conditions it is

quite probable that the lateral movement may have extended to the tips of the piles. It is interesting to note that notwithstanding a movement of about 9 in. this wall remained vertical and did not settle appreciably.

Quite properly, Mr. White calls attention to the "danger of extending observations or calculations made on single piles or on a small group, to a large group, such as is ordinarily done." In some soils under certain conditions such an extension may be entirely satisfactory, whereas for other soils under the same conditions, or for the same soils under other conditions, extrapolation of observations on single piles or a small group to a large group, might result in failure. Mr. White very ably discusses large movements of groups of piles serving as foundations to concrete walls, and the writer is in accord with his suggestion that, in many instances, more positive means of insuring lateral stability by use of struts or ties or other precautionary measures may be highly desirable. The possible effect on structures subjected to lateral loads of vibration such as that resulting from driving piles within a radius of 50 or 75 ft, should be carefully considered, especially when the structure rests on a foundation soil which itself may be affected by vibration.

Professor Krynine has added to the value of the paper by citing and briefly discussing a number of other tests on this interesting subject, including tests made by himself on pile models. He has also given an interesting discussion of the "Theory of the Zero Point." The writer agrees that there is a zero point, but believes that, for piles of the length described in the paper, instead of rising from some point, Z (see Fig. 29), to the ground surface with increasing horizontal load, the zero or pivot point (or point about which rotation occurs, as others refer to it), is nearest the surface with small lateral loads and moves downward with increasing loads. The location of the zero point, which serves to define the bent length of the pile, is regarded by the writer as being a variable which depends on the relationship between the physical properties of the pile (such as size, shape, elastic modulus, and length) and the elastic properties of the soil, and upon the magnitude of the lateral load.

Professor Krynine suggests that in Fig. 32(c) the line, $N M$, is longer than the line, $N M'$; and concludes that the pile is pulled out at Point N, which contradicts the assumption of ideal embedment at that point. The writer agrees that, in order that lateral movement may occur, either the pile must be slightly pulled up, or the top of the pile slightly lowered as it is bent, or the pile must be elongated. That there is a tendency even at low loads for the pile to be pulled up is believed to be clearly indicated by the extensometer readings described in the paper. These readings indicated that the fiber stress in the compression side was only about 60% of that in the tension side (see Fig. 13). Furthermore, in the final test on Monolith No. 6, with an extreme load of about 40 tons per pile, the two concrete piles nearest the jack were actually pulled up about 3 in. whereas the lateral movement was about 6 in. For all practical purposes, however, within the range of lateral loads normally considered in designs, it is believed that it may be assumed that the lower part of the pile is fixed.

The writer agrees heartily with Professor Krynine that it is very desirable that the scope of the tests described in the paper be extended, particularly to other types of soils and other piling arrangements, including piles battered in both directions.

In conclusion, the writer wishes again to call attention to the fact that the conclusions given in the paper were confined to the soil conditions and piling arrangements under which the tests were made. He also wishes to thank those who have participated in the discussions for their contributions to this interesting subject.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WIND STRESSES IN REINFORCED CONCRETE ARCH BRIDGES

Discussion

BY A. A. EREMIN, ASSOC. M. AM. SOC. C. E.

A. A. EREMIN,[23] ASSOC. M. AM. SOC. C. E. (by letter).[23a]—An interesting method of computing wind stresses in arch bridges, devised by Professor Maurice Levy[7], was described by Mr. Blog who, in discussing its various advantages, has not shown its serious limitations. Equations for wind stresses developed by Professor Levy may be applied only to a single arch rib. In the numerical example of wind-stress computations in an arch bridge with two steel ribs, braced with San Andreas cross-braces, as solved by Professor Levy, it was assumed that the bracings were absolutely stiff and two arch ribs were considered as a single arch rib. A similar method of analyzing wind stresses in reinforced concrete arch ribs with elastic braces, such as that shown in Fig. 3, would involve a serious error.

The graphical construction for computing wind stresses in a symmetrical arch rib shown by Mr. Blog, was devised by Professor Levy for a case in which the product of the moment of inertia of the arch rib and the cosine of the angle between a tangent to the arch axis and the horizontal, is a constant. A similar graphical construction of moments in the arch rib in Fig. 3 would require complicated computations for locating the line, u-v, in Fig. 4. In this case, analytical computations are much shorter. With a mechanical calculating machine the coefficients in Table 1 may be computed in few hours. Furthermore, they may be computed with a slide-rule with reasonable accuracy for practical purposes.

Mr. Blog could not understand why the torsion moment, m_v, contributes anything to the value of θ, or that the bending moment, m_u contributes anything to the value of τ. This may be proved with reference to the similarity

NOTE.—The paper by A. A. Eremin Assoc. M. Am. Soc. C. E., was published in December, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1936, by Leon Blog, Assoc. M. Am. Soc. C. E.; August, 1936, by Messrs. Fang-Yin Tsai, and Paul Andersen; and September, 1936, by Louis Blume, Esq.

[23] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[23a] Received by the Secretary November 4, 1936.

[7] "La Statique Graphique", Pt. III, by Maurice Levy.

between forces and deformations in the plane of the arch rib. Vertical forces in this plane produce vertical and horizontal displacements. Likewise, horizontal forces also produce vertical and horizontal displacements. Therefore, Equations (32) and (33) are incorrect if applied to arch ribs.

In evaluating the coefficients, C_{1m} and C_{1t}, Mr. Blog erroneously considers the angle, ϕ, as a variable, which precludes the possibility of verifying the writer's equations. Contrary to Mr. Blog, wind pressures do have an effect on direct stresses in arch ribs. Such stresses in Equation (2) were not considered because they are negligible. In straight beams with ends fixed, which sustain transverse loads the direct stresses are never considered. However, it does not indicate that, in a beam completely fixed at the ends, transverse forces have absolutely no effect on direct stresses. Mr. Blog cites the writer's assumption that, in the braced arch ribs in Fig. 3, both arch ribs resist the same amount of wind pressure, stating that this condition penalizes the system severely. The writer does not agree since, by means of rigid braces, wind forces from the windward rib are transferred to the leeward rib in such a manner as to involve both arch ribs equally. For all practical purposes this assumption is correct and facilitates the location of the points of contraflexure in the braces.

Professor Tsai states that in developing Equations (3) and (4) by means of Castigliano's theorem, the writer failed to show all the steps. This theorem is described in standard textbooks on strengths of materials[24] sufficiently to reveal all the steps between Equations (2) and (3). The method involving the idea of a "dummy" unit force at a displacement point also follows from Castigliano's theorem.

The values in Table 1 were computed by the method of approximate summation. The arch rib was divided into voussoirs as shown in Fig. 3. The physical properties of the voussoirs were computed at their centers of gravity. The limits of integration are shown in Table 1; and, for the half arch rib, the limits are from the springing to the crown section.

Mr. Andersen's Equation (34) is similar to Equation (5) developed for a single arch rib. It is to be noted that, in the braced arch rib shown in Fig. 3, Equation (34) cannot be used for computing the bending moment at the springing. From Equations (5) and (6) the bending moment, M_B, and the torsion moment M_T, at the springing, in a single arch rib, are:

$$M_B = -\int w\, t\, u\, ds + M_w \cos \alpha \qquad (39)$$

and,

$$M_T = +\int w\, t\, v\, ds - M_w \sin \alpha \qquad (40)$$

The bending moment, M_{BS}, and the torsion moment, M_{TS}, at the springing of the braced arch rib in Fig. 3 are:

$$M_{BS} = -\int w\, t\, u\, ds + M_c \cos \alpha - R_1\, a \sin \alpha + R_2\, a \cos \alpha \qquad (41)$$

[24] Interesting information regarding Castigliano's theorem and the originality of Maxwell's theorem may be found in "Strength of Materials", Pt. I, by S. Timoshenko.

and,

$$M_{TS} = + \int w\ t\ v\ ds - M_c \sin \alpha + R_1\, a \cos \alpha - R_2\, a \sin \alpha \ldots.(42)$$

Equations (39) to (42) may be used for computing wind stresses in filled or spandrel open arch bridges such as that shown in Fig. 5, as stated by Mr.

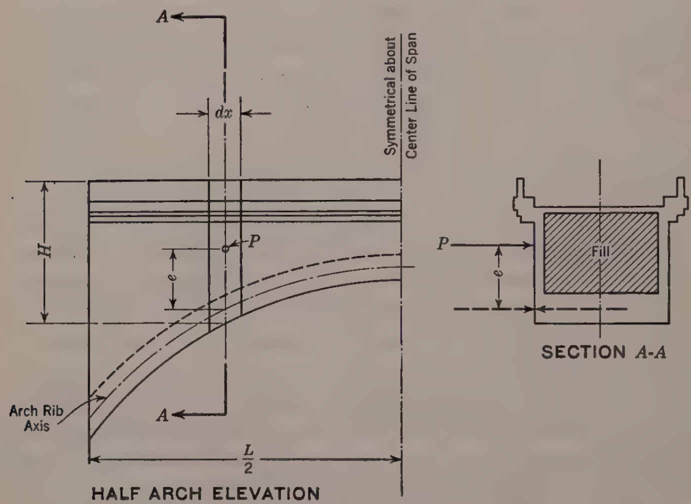

FIG. 6.—SPANDREL FILLED ARCH BRIDGE.

Andersen. Fig. 6 shows a symmetrical spandrel filled arch bridge, subjected to a wind pressure, P, on a vertical strip, dx. In this case,

$$P = w\ H\ dx \ldots\ldots.(43)$$

The force, P, is concentrated at the center of the strip but it may be transferred to the arch axis as a force, P, and an overturning moment, thus:

$$M_P = P\ e = w\ H\ e\ dx \ldots\ldots.(44)$$

Therefore, the wind stresses in the arch rib in Fig. 6 are equal to the sum of the stresses computed with concentrated forces, P, acting along the arch axis, and the overturning moments, M_P. Wind stresses and deformations produced by Force P may be computed by means of Equations (1) to (13). The deformations and stresses in an arch rib sustaining overturning moments, M_P, may be developed by means of Equations (3) and (4).

The moments, m_u and m_v, in the arch rib due to an overturning moment, M_P, are:

$$m_u = - \int M_P \sin \phi\ ds + M_{mw} \cos \phi \ldots\ldots.(45)$$

and,

$$m_v = + \int M_P \cos \phi\ ds + M_{mw} \sin \phi \ldots\ldots.(46)$$

in which M_{mw} is the bending moment at the crown due to the symmetrical overturning moments, M_P. From Equations (45) and (46) and Equations (7) to (10), the deformations in an arch rib sustaining symmetrical moments, M_P, are found to be:

$$\theta_{mw} = -\int \left[\cos\phi \int M_P \sin\phi \, ds + \gamma \sin\phi \int M_P \cos\phi \, ds \right] dw$$
$$+ M_{mw} \int (\cos^2\phi + \gamma \sin^2\phi) \, dw \ldots\ldots\ldots\ldots\ldots (47)$$

and,

$$\tau_{mw} = \int \left[\sin\phi \int M_P \sin\phi \, ds - \gamma \cos\phi \int M_P \cos\phi \, ds \right] dw$$
$$+ M_{mw} \int (I - \gamma) \sin\phi \cos\phi \, dw \ldots\ldots\ldots\ldots\ldots (48)$$

In an arch rib with symmetrical wind pressures, the deformation, θ_{mw}, at the crown is equal to zero. Therefore, from Equation (47), the bending moment at the crown is:

$$M_{mw} = \frac{\int \left[\cos\phi \int M_P \sin\phi \, ds + \gamma \sin\phi \int M_P \cos\phi \, ds \right] dw}{\int (\cos^2\phi + \gamma \sin^2\phi) \, dw} \ldots (49)$$

For integrating Equations (47) to (49), the overturning moment, M_P, should be expressed as a continuous function. However, in practice the integration may be performed as an approximate summation without serious error. The arch rib is divided into a convenient number of vertical strips and forces, P, with overturning moments, M_P, computed for each strip. Strips that intersect the arch rib are considered as arch voussoirs and are treated in the manner shown in the illustrative example given in the paper (see Fig. 3).

Evidently, in the summation, the wind forces, P, the moments, M_P, and the physical properties of an arch rib must be considered at the centers of gravity of the voussoirs. An approximate method of summation is especially convenient in open spandrel arch bridges, in which wind-stress pressures vary abruptly along the arch axis, at the spandrel column.

Mr. Blume has shown the intermediate steps in developing Equation (13) from Equation (2). However, he was incorrect in considering the ratio, γ, to be constant. Obviously, it varies, along the arch axis, with the moment of inertia, I, and the torsion factor, F.

In closing, the writer wishes to express his appreciation for several interesting contributions to the discussion.

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DISCUSSIONS

ADMINISTRATIVE CONTROL OF UNDERGROUND WATER: PHYSICAL AND LEGAL ASPECTS

Discussion

By Messrs. H. J. F. Gourley, and O. J. Baldwin

H. J. F. Gourley,[12] M. Am. Soc. C. E. (by letter).[12a]—The subject-matter of this paper is of great interest, if only as an indication of the variation in the practice in different parts of the United States, and forms a valuable record. To complete the picture, the following notes of the position in England are submitted.[13]

In 1859, the highest legal tribunal—the House of Lords—established the principle that the rights of riparian owners, in respect of water flowing in visible surface streams or in known and defined channels, do not apply to underground water which simply percolates into and through the ground and has neither certain course nor defined limits. Subsequent cases decided in 1881, 1886, and 1902, made it clear that a "defined" channel is one which is definitely contracted and bounded, although the actual course of a channel may be undefined by human knowledge, and, furthermore, that a defined subterranean channel must be "known" not by excavation but by reasonable inference from observations on the surface of the ground.

By these decisions an owner claiming riparian rights in the flow of water in a subterranean channel must prove the existence of such a channel without recourse to excavation, and as recently as 1932 this statement of the legal position was affirmed by the Court. The general position, therefore, is that there is no property right in underground water, and nothing to prevent the sinking of a well on land adjacent to the site of an existing well, to the possible detriment of that well.

Note.—The paper by Harold Conkling, M. Am. Soc. C. E., was published in April, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1936, by Messrs. Joseph Jacobs, and W. D. Faucette and J. E. Willoughby; and September, 1936, by R. E. Savage, Assoc. M. Am. Soc. C. E.

[12] Civ. Engr. (Binnie, Deacon & Gourley), Westminster, London, England.

[12a] Received by the Secretary October 23, 1936.

[13] See "Recent Developments in British Waterworks Practice", by H. J. F. Gourley, *Journal*, New England Water Works Assoc. (Publication pending.)

As between neighboring industrial concerns using underground water, the principal effect of the existing law has been that when "Peter" robbed "Paul" by sinking to, and pumping from, a greater depth than "Paul", the latter's only remedy was to deepen his own well and pump from still greater depths, and the only people who gained were the well sinkers.

Until comparatively recently an industrial concern using more than 100 000 gal of water per day from an underground source was the exception; within the past decade, artificial silk and other specialized works have been established, and these, by drawing on underground resources to the extent of several million gallons per day in some cases, have so depleted (over-pumped) the area in which the works happen to be, as to affect adversely the yield of public wells upon which the community relies for all or part of its supply.

It will have been appreciated that a private individual or manufacturing concern has only to buy land in order to have the indisputable right to obtain such water from underground sources as the strata are capable of yielding.

When, however, a public authority or statutory water undertaker[14] contemplates the augmentation of its resources by an underground supply, the position is not so simple, since the authority of Parliament or of the Ministry of Health must be obtained first. Parliamentary plans are then deposited in both Houses (that is, the House of Lords and the House of Commons), in other specified places, and with Government departments. A Bill is prepared describing the "works" for which powers are sought, and among its other provisions is a so-called "protective clause." At the same time, by notices published in the newspapers, all persons or interests likely to be concerned are made aware of the proposals of the Bill, which is, in due course, the subject of inquiry, in turn, by a committee of each House. The promoters and the various opposing interests are represented by counsel and supported by engineers, and other technical witnesses, and, in the sequel, the Bill emerges as an Act upon receiving the Royal Assent, although, generally, with amendments put upon the promoters by one or other committee or inserted by agreement with opponents. It is by such an Act that a public authority is empowered to purchase land by condemnation, if agreement is not possible, and to sink a well or wells, and to build other structures, on it. For less important works and where there is no opposition, a simpler and less expensive procedure is available, for the Minister of Health has authority to grant a Provisional Order, which must subsequently be confirmed, if not then opposed, by Parliament.

For thirty years or more most private Acts involving the construction of wells have included the protective section previously mentioned. Under this section, the owner of any well or spring (and, sometimes, pond) within a specified radius, used at the date of the Act as a source of water supply, who can prove that his supply has been diminished as a consequence of the pumping at the authorized well, is entitled to compensation in water, or otherwise. The radius of protection varies with the geological formation and in special

[14] Throughout this discussion the term, "undertaker", means, according to the context, a municipal authority or a public utility company empowered by statutory authority to construct and operate works for supplying water within a specified area.

circumstances, but it is usually 1 mile for wells in chalk and 2 miles for wells in Triassic sandstone. It also sometimes happens that, having regard to representations made during the Parliamentary proceedings on behalf of opposing statutory water undertakers, a limit is put on the quantity that may be pumped in any day.

Thus, it will be seen that, whereas private individuals have been unfettered in their exploitation of underground water resources, those charged with the duty of affording supplies to the community have, by precedent and practice, been restricted as it were outside the general law; and until 1936 there was no case of reciprocal protection being given to the public authority and nothing in law to prevent an industrial concern, with the knowledge that an authorized well had proved the presence of a large supply, from sinking its well immediately adjacent to the authorized well and developing it so as seriously and adversely to affect the yield of the authorized well.

Partly to meet the growing needs of its area of supply and partly because one of its authorized wells had had its yield reduced from 1 500 000 to 700 000 gal per day in the course of eight years, the Wolverhampton Corporation sought Parliamentary powers this year (1936) to sink additional wells at some distance from the Borough. The reduction in yield was attributed to the pumping of increasingly large quantities (now about 2 000 000 gal per day) from the same Triassic formation by an industrial concern the works of which are within the Borough and less than a mile from the well; and evidence was given to the effect that, in the course of a few more years, the yield of that well would become so small as not to be worth pumping by the Corporation.

Although the Bill, which included the usual protective clause with a radius of 2 miles, was opposed, it emerged with a reciprocal clause by which, after the date of the Royal Assent, no land-owner within the same radius could sink a well or otherwise abstract underground water, except by consent of the Corporation, except for the domestic supply of persons on his land, for the watering of stock, or for other purposes of an agricultural nature connected with the product of that land. It was not the intention of the Corporation to impose undue restrictions on the normal development of the land within the 2-mile radius, and the clause was definitely directed to preventing the incursion of large industrial works. The clause provided that on refusal by the Corporation of consent to sink a well, the land-owner may require the Corporation to afford a supply at cost price to the Corporation, although there is no obligation if by so doing there is likely to be interference with the Corporation's ability to maintain either the supply for domestic purposes within its statutory limits of supply, or with existing trade supplies, nor if the aggregate of such supplies, together with what water was necessary to compensate owners of existing sources within the radius of protection, exceeds the yield of the authorized well.

Having regard to the extent to which underground resources were being exploited without control, water-works undertakers for many years have been pressing for general legislation to control the development of under-

ground sources, and advocating a system of licensing by which regard would be had to the limits of potential yield of particular underground basins; and during the past two or three years the Institution of Water Engineers of Great Britain, acting jointly with other associations representing water-works interests, has made representations to the Minister of Health which were supported by him in a memorandum placed before a Joint Committee of both Houses of Parliament which sat at intervals during 1935 and 1936. This Committee reported on July 29, 1936, and, after referring to the anomalous position of water supply undertakers as compared with private individuals (as previously described), states that "it would be more equitable and more in the public interest if property owners were treated in the same way as water undertakers in any project where the former may wish either to sink wells below a certain depth or extract more than a certain quantity of water from them except for their own agricultural or domestic purposes." It remains to be seen how far general legislation follows this recommendation; it is sufficient to observe that the Wolverhampton Act is in advance of the general law as it now stands, and since, in the absence of general legislation, Parliamentary Committees on Private Bills are more often than not guided by precedent, this Act will serve as an argument in favor of similar provisions in future Bills.

So far as regional considerations affect underground-water resources and supplies, the problem is by no means as simple and as capable of reasonably clear-cut limitations as in the case of surface supplies. To appraise with any reasonable and approximate degree of accuracy the limits of a particular underground basin, its rate of replenishment, its storage capacity, and its potential yield, is a difficult and often involved undertaking. In most cases it is only after years of exploitation that it becomes apparent, on an investigation and consideration of ground-water levels, on the character of the water yielded, or on a combination of these factors, that water is being extracted in excess of replenishment. Often, particularly in areas adjacent to the sea, it is too late to remedy a partial or complete spoilation of an otherwise potable supply which has resulted from over-pumping in such circumstances.

What has been stated herein, indicates that there is now a tendency in England toward what the author describes as the "American rule" and also to what may be considered as a measure of State control. In so complicated a matter, the step-by-step approach, as the author observes, is probably the safest line and one which will allow of re-orientation in the light of experience.

O. J. BALDWIN,[15] JUN AM. SOC. C. E. (by letter).[15a]—The able manner in which this paper has been presented is a subject for congratulation. The value of administrative control of underground waters is well emphasized and a careful study of the paper would be of great value to all concerned with the utilization and conservation of this natural resource.

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[15] Chf. Engr., The Iowa State Planning Board, Ames, Iowa.

[15a] Received by the Secretary November 9, 1936.

There are other aspects of the use of underground water, not mentioned by Mr. Conkling, which are of major importance in many States, particularly in the humid sections in the Mid-Western and Eastern United States where irrigation is not an important water use. In this region the demands of business and industry upon underground water for air-conditioning and industrial use have placed drafts of such magnitude on small areas that static levels are being, and have been, seriously lowered. The continued exploitation of underground waters will result in conditions which, undoubtedly, will produce a fifth doctrine of law, or a modification of the fourth doctrine discussed by the author.

Specifically, the point that must be decided by the Courts, or written into future legislation, concerns the proposed use of the water. Shall the municipal or domestic consumption of underground waters be considered a more basic or fundamental use than those of industry (manufacturing), refrigeration, and air-conditioning? If so, what shall be the priority of use, and shall a prior claim on the underground waters be subject to condemnation proceedings to meet the needs of a more fundamental use, as is the case with surface waters in some States? Inasmuch as air-conditioning and refrigeration may be obtained by other, although more expensive, means it would appear that the development of sub-surface waters for these purposes should be definitely controlled by legislative action.

Legislative control of air-conditioning and cooling usage should not be construed as a prohibition of such usage. Where the cooling effect is obtained by circulation through a closed system, the water may be returned to the source, through recharge wells, without changing the mineral or bacterial content. This method of maintaining the static level has already been used successfully in some of the Eastern metropolitan areas.

The use of a recharge well at a point removed from the supply well is more expensive to the individual user than wasting the water into sanitary sewers. On the other hand, the overloading of the sanitary sewerage system and the sewage disposal plant of a municipality with waste water from air-conditioning plants will greatly increase the cost of operation and maintenance. Already, some cities have found legislative action necessary to force the operators of air-conditioning plants to cease overloading sanitary sewers by connecting to the storm sewers. It is not unlikely that a continued development of sub-surface waters (provided the supply is not exhausted) may lead to an overloading of storm sewers also. Even if the storm sewers prove adequate to carry the flow, the wasteful use of artesian water is not halted. Viewed in the light of conserving the underground waters, any reasonable increase in expense, incurred in returning the waste water from closed-circuit air-conditioning plants to the ground, can be justified.

The use of a recharge well presents certain problems aside from those of expense. Foremost of these are the questions of minimum spacing between wells to prevent recirculating the returned water, and the temperature rise to the return of the heated water. The interval between recharge and supply wells must be determined by studies of the individual cases, giving due weight

to the porosity of the aquifer, the movements of the artesian stream, the location of adjacent wells, and other factors peculiar to the site.

A rise of only a few degrees in the temperature can materially lower the efficiency of a refrigerating or air-conditioning system. To offset the temperature rise, it seems entirely possible, from the practical aspects, to utilize a closed system with a recharge well both as a cooling system in the summer and a heating plant in the winter. With an underground water temperature of 50 to 60° it would be possible to use the cooling system as a heating plant probably 20% of the year. A temperature differential of only a few degrees would permit a material saving in fuel costs in heating fresh air brought into the building, whereas warehouses might be completely heated to a satisfactory temperature.

Temperature differences of 75° would be quite common for short periods in the Northern United States. Under such conditions it is possible that sufficient "cold" could be stored in the sub-surface formations to eliminate the temperature rise of the following summer. It is quite likely that an actual economy could be effected, over a yearly operation of individual heating and air-conditioning plants, that would more than offset the added expense of a closed system. Regardless of the comparative cost, however, a less wasteful usage of sub-surface water resources will be one of the conservation problems of major importance in the near future.

The need of wisely drafted legislation, based on a thorough study of the water resources, is most urgent. The Engineering Profession is perhaps as responsible as any one group for the uncontrolled exploitation of this natural resource. It is now its responsibility to encourage the conservation of artesian waters and to lend its support to obtaining legislative appropriations for studies of the many problems relating to sub-surface waters. The tempo of the present development calls for an immediate study of extensive proportions in order that control measures may be adopted before depletion of the underground supply occurs. The writer is in accord with the conclusions that the control of undergraund water should be retained in the State. The complex situations existing in the various States make legislative control sufficiently difficult even within the State. Only in special situations where the interest of more than one State is involved, should there be any attempt at regulation by higher authority.

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DISCUSSIONS

SIMULTANEOUS EQUATIONS IN MECHANICS SOLVED BY ITERATION

Discussion

By M. B. Gamet, Jun. Am. Soc. C. E.

M. B. Gamet,[11] Jun. Am. Soc. C. E. (by letter).[11a]—An interesting terminology to apply to methods of solution for, and types of, equations is developed in this paper. The writer wishes to commend highly the use of the term, "iteration", as a substitute for "successive approximation."

The writer wishes to make some very important additions to the author's "List of References" in Appendix I of the paper. These additions[12] represent the pioneer work done in the United States with this so-called "iteration" procedure in the solution of simultaneous equations of the type discussed by the author.

It will be noted that in Appendix I the author has listed no references to any of the numerous American developments in this field which occurred prior to 1930. All the references which he cites refer to German sources prior to 1930, whereas the method was well developed in the United States by that time. .

Another important change in terminology is the use of the expressions, the "Equation of Three Angles", the "Equation of Five Angles: No Side-Sway", and the "Equation of Five Angles with Side-Sway", as a series of substitutes for the term, "slope-deflection" equations. Evidently, this terminology originates with the author's reference to the work of W. Gehler (3)[3] published in Germany in 1925.

There seems to be a continual need to remind recent students of the use of angular joint rotations as unknowns in rigid-frame problems, of the facts

Note.—The paper by W. L. Schwalbe, Esq., was published in August, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Garrett B. Drummond, and A. W. Fischer.

[11] Instr. in Civ. Eng., School of Eng., Northwestern Univ., Evanston, Ill.

[11a] Received by the Secretary, October 30, 1936.

[12] "Secondary Stresses in the Kenova Bridge", by Allston Dana, M. Am. Soc. C. E., *Engineering News*, 1913; "Statically Indeterminate Stresses", by John I. Parcel, and George A. Maney, Members, Am. Soc. C. E., Wiley & Sons, 1936, Fig. 142, p. 276; "Investigation of Secondary Stresses in the Kenova Bridge," by John I. Parcel and George A. Maney, Members, Am. Soc. C. E., *Studies in Engineering, No. 4*, Univ. of Minnesota, 1922; and by G. A. Maney, M. Am. Soc. C. E., *Studies in Engineering, No. 1*, Univ. of Minnesota, 1915, p. 17.

[3] Numerals in parentheses refer to Appendix I of the paper.

in the history of the development of this useful method of approach. As early as 1880, Manderla made use of angular rotations of joints in a rigid frame structure to determine the secondary-stress bending moments in a bridge truss. Soon after this (1893) Mohr used a similar approach which was identical with that which the slope-deflection method makes to the "general" problem of the rigid frame. Before 1915, only the isolated problem of secondary stresses had been approached in this manner.

In 1915, G. A. Maney, M. Am. Soc. C. E. published the first general statement of the slope-deflection method in this country[13]. In this treatment, not only are secondary stresses solved, but wind stresses and all types of frames with members transversely loaded as well. The idea of fixed-beam moments was there first developed, although at the time it was not called by that name. This work led immediately to the important publications originating at the University of Illinois.

In 1914, Professor Maney and W. M. Wilson, M. Am. Soc. C. E., developed, and in June, 1915, published[14] the solution of wind stresses in a twenty-story office building by the slope-deflection method. This solution was by the exact simultaneous method. In 1918, Professor Wilson and F. E. Richart, and C. Weiss, Members, Am. Soc. C. E., published a bulletin[15] giving general formulas for a large number of types of structures by the slope-deflection method. These two publications, following Professor Maney's publication and using his methods, gave the "angular unknowns" approach to rigid frame and continuous beam solutions for bending moment distribution the great popularity it has had in the United States.

It should be emphasized that this is a typically American development and that the "general" use of the method of "angle unknowns" has lagged considerably in Europe. (This statement excepts only the problem of secondary stresses which was mentioned by Professor Maney[14].) The European approach to the general case of the rigid frame by the "unknown angles" method is outlined as follows: In May, 1914, Axel Bendixsen[16] published, in Denmark, a monograph which gives the first general statement of the slope-deflection relation for any type of loading. This publication appears to have been generally overlooked even in Europe until about 1923 when Professor A. Ostenfeld amplified it somewhat. In 1923, Professor Ostenfeld published[17] an amplification of Bendixsen's work, the presentation covering all cases for deformations rather than stresses.

An unusual development has followed Professor Maney's general statement of the use of angles as unknowns. Numerous ideas such as moment distribution, and the balancing of angle changes, have followed in its wake and they have all been strictly American variations of the "slope-deflection" theme.

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[13] *Studies in Engineering, No. 1*, Univ. of Minnesota, 1915.

[14] "Wind Stresses in Steel Frames of Office Buildings", by William M. Wilson, and George A. Maney, *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1915.

[15] "Analyses of Statically Indeterminate Structures by the Slope-Deflection Method", by William M. Wilson, F. E. Richart, and Camillo Weiss, *Bulletin No. 2*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1918.

[16] "Die Methode der Alpha-Gleichungen zur Berechnung von Rahmenkonstructionen", von Axel Bendixsen, May, 1914.

[17] "Die Deformationsmethode", by A. Ostenfeld, *Der Bauingenieur*, January 31, 1923.

Although this general idea of treating angles as unknowns was first presented in the United States by Professor Maney in 1915, it has since almost completely replaced the "least work" and "virtual work" approach to the problem. In Europe, these latter methods are still much used, in spite of their greater mathematically cumbersome nature.

In the following development some of the important equations and methods referred to by the author may be traced.

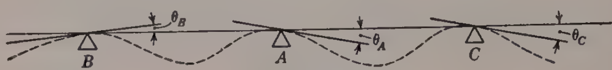

FIG. 15

Item (1).—In Fig. 15,

$$M_{AB} = \pm M_{F(AB)} + 2E\,K_{AB}\,(3\,R_{AB} - 2\,\theta_A - \theta_B)\,\ldots\ldots\ldots(58)$$

is the general form of the slope-deflection equation stated by Professor Maney (13) in 1913 If two bending moments are equal and opposite in sign, as occur at each support of a continuous beam: $M_{AB} + M_{AC} = 0$; and, when $R = 0$,

$$M_{AB} = \pm\, M_{F(AB)} - 2\,E\,K_{AB}\,(2\,\theta_A + \theta_B)\,\ldots\ldots\ldots\ldots(59)$$

and,

$$M_{AC} = \pm\, M_{F(AC)} - 2\,E\,K_{AC}\,(2\,\theta_A + \theta_C)\,\ldots\ldots\ldots\ldots(60)$$

Therefore,

$$K_{AB}\,\theta_B + 2\,(K_{AB} + K_{AC})\,\theta_A + 2\,K_{AC}\,\theta_C = \frac{\pm\, M_{F(AC)} \pm M_{F(AB)}}{2\,E}\ldots(61)$$

It will be seen that Equation (61) is identical with Equation (28) of the paper and is merely the slope-deflection "joint" equation for continuous beams. This is the equation which Gehler (3) called the "equation of three angles" in 1925, and has led to the author's use of this terminology. Similarly, the "equation of five angles" is merely a slope-deflection joint equation for the typical case of a rigid frame with four members framing into a rigid joint, instead of two, as in the preceding case of a continuous beam.

Item (2).—When the factor, $2E$, is cancelled and θ_B and θ_C are assumed to be zero temporarily, then Equation (61) becomes: $\theta_A = \dfrac{\pm M_{F(AC)} \pm M_{F(AB)}}{2\,(K_{AC} + K_{AB})}$; and, in general, $\theta_A = \dfrac{\Sigma\, M_{F(A)}}{2\,\Sigma\, K_A}$ if there are two or more members at the joint, and $\Sigma\, M_{F(A)}$ = the algebraic sum of M_F-values at Joint A whereas $\Sigma\, K_A$ = the numerical sum of the K-values $\left(K = \dfrac{I}{L}\right)$ around Joint A.

Similarly, $\theta_B = \frac{\Sigma M_{F(B)}}{2 \Sigma K_B}$. If these expressions for θ_A and θ_B are substituted in Equation (61):

$$M_{AB} = \pm M_{F(AB)} - \frac{\Sigma M_{F(A)}}{\Sigma K_A} - \frac{1}{2}\left(\frac{\Sigma M_{F(B)}}{\Sigma K_{(B)}}\right) \quad \text{.........(62)}$$

Equation (62) is simply the "moment distribution" equation in which $\frac{\Sigma M_{F(A)}}{\Sigma K_A}$ is the "distribution" factor and $\frac{1}{2}\left(\frac{\Sigma M_{F(B)}}{\Sigma K_B}\right)$ is the "carry-over" factor. By this variant of the slope-deflection method, the "iteration" method is applied through this "carry-over" factor until error is negligible.

Conclusion.—It is important to remember in connection with the entire general plan of solution by means of "angle unknowns", that the M_F-value, or the "fixed-beam" moment is the essential starting point, and that until the slope-deflection method was evolved in 1913, no one had previously used this approach. In connection with the problem of secondary stress, this conception was not involved.

Why do all equations of the type discussed by the author invariably converge when the so-called "iteration" process or "converging approximations" is used? The answer is found to be a purely physical reason which will only be confounded by any mathematical approach. If Equation (59) is examined it reveals that the angle change, θ_A, at the joint in question, has twice as much influence on the final moment, M_{AB}, as the angle change, θ_B, at the far end from the joint in question. Furthermore, the last term of Equation (62), or the "carry-over" factor, reveals the same thing. When an error at any joint adjacent to the one for which the angle or moment is desired is always at least twice the error caused at the joint in question, rapid convergence by halving of the error at each iteration is assured.

The simple physical facts make convergence, or solution of equations by iteration, possible. If the quantitative importance of angle changes at each end of a member were nearly the same in affecting the moments at both ends, then "successive convergence" or "iteration" would have no place as a method of solution for moments in continuous-beam and rigid-frame members.

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DISCUSSIONS

ANALYSIS OF CONTINUOUS FRAMES BY BALANCING ANGLE CHANGES

Discussion

By Messrs. John E. Goldberg, G. A. Maney, Paul Andersen, and William F. Luce

John E. Goldberg,[8] Jun. Am. Soc. C. E. (by letter).[8a]—The method of analyzing continuous frames by balancing end rotations presented by Professor Grinter is similar in many respects to the simple slope-deflection method largely developed by G. A. Maney, M. Am. Soc. C. E., from his basic slope-deflection equations and presented[9] by the writer in 1931. Both methods consist, essentially, of determining the angular positions at which the various joints of the continuous structure are in equilibrium, or balanced, and then substituting these values of the angular rotations in the basic slope-deflection equation to obtain the final or actual moments at the ends of the various members. Both methods may be extended, by the application of well-known and well-understood principles, to the solution of a wide variety of rigid-frame problems. The advantages which Professor Grinter cites for his method are at least as applicable to the earlier slope-deflection method.

The slope-deflection method seems, to the writer, to have certain additional inherent advantages; for example, the starting point of the slope-deflection method is the fixed-beam moments which are again used to obtain the final or actual moments after the various rotations have been determined. This itself may be construed as a twofold advantage: First, a complete series of calculations (the simple beam end-angles) is avoided; and, second, the fixed-beam moments are themselves better known and more easily remembered for the various standard loadings than the simple beam end-angles. Another advantage lies in the fact that, throughout the entire slope-deflection analy-

Note.—The paper by L. E. Grinter, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[8] Chicago, Ill.

[8a] Received by the Secretary October 19, 1936.

[9] "Vertical Load Analysis of Rigid Building Frames", by John E. Goldberg, *Engineering News-Record*, November, 12, 1931; see, also, "Simplified Methods for the Analysis of Multiple Joint Rigid Frames", by George A. Maney and John E. Goldberg, *Northwestern Univ. Bulletin*, Vol. XXXIII, No. 7, October 17, 1932.

sis, the concept of continuity is firmly maintained, a fact that adds greatly to the clarity and physical significance of the method, besides being inherently more direct. In Professor Grinter's analysis, on the other hand, this continuity is first completely destroyed and then successive parts of it are re-established. Another inherent advantage of slope-deflection methods lies in the fact that there are frames the analysis of which by methods based upon successive approximations, corrections, or balancing, may be inadvisable, impractical, or even impossible.

Various types of simple frames, for example, may be analyzed by slope deflection by the solution of a limited number of equations, in many cases by only one equation. Simple bridges, culverts, and many other similar frames fall into this category of frames which may be analyzed with actually less effort and greater reliability by direct methods. The frame shown in Fig. 2, although chosen by the author for its classical values, is nevertheless an illustration of a structure that could be solved very neatly by these direct slope-deflection methods. One who is adept and familiar with the slope-deflection theory could very easily set up the two equations which would solve the frame. Solution of these equations would give, with absolute certainty, the joint rotations from which the final or actual moments may be calculated. There are also frames which, by their very proportions, can not be solved successfully by methods based upon successive approximations or corrections. For the analysis of these frames there always remains the alternative method of solving the slope-deflection equations by algebra.

Professor Grinter, it seems, has touched upon the matter of bent analysis too lightly. He implies that if the shear for any assumed side-sway at a particular story is known, the actual shear in the various members may be calculated by a single, simple, proportionate adjustment. Unfortunately, the analysis is not so simple, except in a very few mathematically perfect cases, chief among which is the single-story bent, itself not a very complex problem. Equations (4) and (5), the slope-deflection equations, show that the shear in a member is as much a function of the joint rotations at the ends of the member as of the relative joint translation. These rotations, in turn, are affected just as much by the shears above and below the story under consideration as they are by the shears in that particular story, etc. Consequently, except in these few perfect cases, this simple adjustment is itself only an approximation.

Professor Grinter has referred to the method of analyzing wind stresses by slope-deflection and converging approximations, which the writer developed from basic slope-deflection theory and presented[10] in 1933, as being based upon series convergence. Actually, the method consists simply of obtaining, successively, closer or more nearly correct values of the various rotations in accordance with simple and easily understood principles, and thus to label the method with an abstract mathematical term is confusing to the student and to the reader. Furthermore, he has referred to the method as being "relatively tedious" without, however, stating to what it is relative. Never-

[10] "Wind Stresses by Slope-Deflection and Converging Approximations", by John E. Goldberg, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 962–975.

theless, after an exhaustive survey of methods of wind stress analysis, the results of which are contained in its latest report[11], Sub-Committee No. 31, Committee on Steel of the Structural Division of the Society, on Wind-Bracing in Steel Buildings points out that among the desirable characteristics of the writer's method are speed, accuracy, simplicity, and the clarity of the physical concept maintained throughout the analysis. Professor Grinter's present appreciation of the slope-deflection equation and of the importance and significance of the physical concept of joint rotations is definitely complimentary to the slope-deflection method and to those who have consistently sponsored and developed its use.

The frame of Fig. 2 is analyzed herein by slope deflection to illustrate the general methods of attack. One familiar with the slope-deflection theory can set up, practically by inspection, the joint equations which express the simple physical fact that ΣM (in terms of fixed-beam moments and joint rotations) equals zero. For the frame of Fig 2:

$$\text{Joint } B: 11.0\ \theta_B + 4.0\ \theta_D = 8.333$$

and,

$$\text{Joint } D: 4.0\ \theta_B + 21.5\ \theta_D = -\ 12.0833$$

From which, by simple algebra:

$$\theta_D = \frac{11.0\ (-12.0833)\ - 4.0\ (8.333)}{11.0\ (21.5)\ - 4.0\ (4.0)} = -0.754$$

and,

$$\theta_B = 0.758 + 0.274 = 1.032$$

Substitution of these values in the basic slope-deflection equation,

$$M_{AB} = M_{F\text{-}AB} - K\ (2\,\theta_A + \theta_B) \quad \text{(8)}$$

gives the final end moments; that is, $M_{DB} = -\ 8.333\ - 4.0\ (-1.508 + 1.032) = 6.429$; and, similarly, for the other end moments. The moments are in kip-feet; and, K-values are used in only a relative sense. As a result of these facts, the joint rotations in the foregoing analysis are approximately one-fiftieth of the rotations indicated by Professor Grinter.

When direct analysis of a frame is so simple, there is scant justification for the use of methods based on successive balancings or corrections, or even for the use of methods based on semi-empirical formulas. However, as an illustration of an alternative slope-deflection method, particularly applicable to frames in which the number of joints precludes the use of algebraic methods, the analysis of the frame of Fig. 2 by converging approximations is also given: The respective joint equations are, for simplicity, reduced and solved for the respective values of θ:

$$\text{Joint } D: \theta_D = -\ 0.562 - 0.186\ \theta_B$$

and,

$$\text{Joint } B: \theta_B = 0.758 - 0.364\ \theta_D$$

[11] *Proceedings*, Am. Soc. C. E., March, 1936, p. 397.

For the purpose of obtaining a first approximation, θ_B is taken to be equal to the constant term in its own equation; that is, 0.758. Then, $\theta_D = -0.562 - 0.186\ (0.758) = -0.703$. From which, $\theta_B = 0.758 - 0.364\ (-0.703) = 1.014$.

For a second approximation: $\theta_D = -0.562 - 0.186\ (1.104) = -0.751$; and, $\theta_B = 0.758 - 0.364\ (-0.751) = 1.031$. For all practical purposes, this second approximation is sufficiently accurate. However, a third approximation gives: $\theta_D = -0.562 - 0.186\ ((1.031) = -0.754$; and, $\theta_B = 0.758 - 0.364\ (-0.754) = 1.032$.

By comparison of these results it is clear that convergence is now complete. Substitution of these final θ-values in the respective basic slope-deflection equations gives the final end moments for the various members. It is clear that no appreciable inaccuracy would have resulted if the θ-values obtained as a second approximation were used to calculate the final end moments.

G. A. Maney,[12] M. Am. Soc. C. E. (by letter).[12a]—In comparing the method used in his paper with the moment distribution method, the author comes to two conclusions as follows: (1) It is essentially self-checking; and (2) it

TABLE 1.—Complete Solution of Problems in Fig. 2 and Fig. 9

(a) End Moments in Fig. 2

Line No.	Symbol (1)	Coefficients of: θ_B (2)	Coefficients of: θ_D (3)	Fixed-beam moments, M_F (4)	Value of final moment, in foot-kips (5)
Complete Check Computations for Final End Moments					
1	M_{BA}	—3	....	....	—3.09
2	M_{BD}	—8	—4	+8.33	+3.09
3	M_{DB}	....	—8	—8.33	—6.36
4	M_{DE}	....	—3	....	+2.25
5	M_{DF}	....	—4.5	—3.75	—0.38
6	M_{DC}	....	—6	....	+4.50
7	M_{CB}	....	—3	....	+2.25
...	M_{ED}	....	....	....	+20.00
Converging Solution for Values of $2\,E\,\theta$					
	$E\,Q$ for:				
8	J_B	+11	+4	+8.33†	....
9	J_D	+4	+21.5	—12.08†	....
10*	....	+1.0000	0.364	....	....
11*	....	+0.186	1.000	....	....
12*	....	+0.758	—0.566	....	....
13*	....	+0.205	—0.141	....	....
14*	....	+0.051	—0.038	....	....
15*	....	+0.014	—0.009	....	....
16*	....	+0.003	—0.002	....	....
17	$2\,E\,\theta$	+1.031	—0.756	....	....

(b) End Moments in Fig. 9, Including Effect of Angular Discontinuities

Symbol (6)	Coefficients of: θ_B (7)	Coefficients of: θ_C (8)	Fixed-beam moments M_F (9)	Corrections due to the angles, ϕ (10)
Moment Equations				
M_{BA}	—4.98	....	—60.0	—0.996 M_{BA}
M_{BC}	—3.60	—1.80	+10.0	—0.448 M_{BC}
M_{CB}	—1.80	—3.60	—10.0	—0.249 M_{BC}
M_{CD}	....	—2.70	+15.0	....
Final Moment Equations				
M_{BA}	—2.49	....	—30.0	—17.1‡
M_{BC}	—2.40	—1.20	+6.7	+17.1‡
M_{CB}	....	—3.30	—11.7	—10.7‡
M_{CD}	—1.20	—2.70	+15.0	+10.7‡
Converging Solution for Values of $2\,E\,\theta$				
$E\,Q$ for:				
J_B	+4.89	+1.20	—23.3	....
J_C	+1.20	+6.00	+3.3	....
*	+1.00	+0.246	......	....
*	+0.20	+1.00	......	....
*	—4.76	+0.55	......	....
*	—0.14	+0.95	......	....
*	—0.23	+0.03	......	....
*	—0.01	+0.05	......	....
$2\,E\,\theta$	—5.14	+5.18	......	....
......	....	....	......	....
......	....	....	......	....

* Balancing angle changes. † $\Sigma\,M_F$. ‡ Values of final moment, in inch-kips.

[12] Prof. of Structural Eng., School of Eng., Northwestern Univ., Evanston, Ill.

[12a] Received by the Secretary October 25, 1936.

offers the clearest possible picture of the physical action of the structure. The writer agrees heartily with these conclusions and submits four short solutions in tabular form, which are made by using the slope-deflection method first proposed by him in the United States in 1915. These solutions (which explain themselves because of the tabular forms used), together with the past history of slope-deflection and of converging approximate solutions, lead to the following comments:

(1) "Balancing angles changes" is merely a new group of words to describe what has been done for years in the solution of slope-deflection equations by converging approximations of the θ-values.[13]

TABLE 2.—SOLUTION OF PROBLEM IN FIG. 10

Symbols	(a) ASSUMING RIGID JOINTS			(b) ASSUMING FINAL ANGLE SLIPS IN THE JOINTS AND DETERMINING THE REDUCTION IN MOMENTS THUS CAUSED					
	Coefficients of:		Fixed-beam moments, M_F	Coefficients of:		Fixed-beam moments, M_F	Correction due to yielding angles	Value of final moment	Percentage reduction in moment
	θ_B	θ_C		θ_B	θ_C				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
COMPLETE CHECK COMPUTATIONS FOR MOMENTS									
M_{BA}	—5.4	....	—450	—5.4	....	—450	+147	—164†	41
M_{BC}	—7.2	—3.6	+300	—7.2	—3.6	+300	—127	+164†	41
M_{CB}	—3.6	—7.2	—300	—3.6	—7.2	—300	+40	—556†	19
M_{CF}	....	—5.4	....	....	—5.4	....	....	—292†	23
M_{CD}	....	—7.2	+1 950	....	—7.2	+1 950	—421	+1 140†	21
M_{CE}	....	—5.4	....	....	—5.4	....	....	—292†	23
M_{DC}	....	—3.6	—1 950	....	—3.6	—1 950	—210	—2 355†	—7
EXACT SOLUTION FOR VALUES OF $2 E \theta$									
$E\,Q$ for: J_B	+12.6	+3.6	—150*	+12.6	+3.6	—130*	....	....	....
J_C	+3.6	+25.2	+1 650*	+3.6	+25.2	—1 269*	....	....	....
....	—3.6	—1.04	+43	—3.6	—1.04	+37	....	....	....
M_F for:	....	+24.16	+1 693	....	+24.16	+1 306	....	....	....
θ_C	....	....	+70	....	....	+54.0	....	....	....
θ_B	....	....	—32	....	....	—25.7	....	....	....

* ΣM_F. † Inch-kips.

(2) In the problems of Tables 1 and 2 values of $2 E \theta$ were determined by the usual method of "balancing angles" or "converging slope values", instead of the θ-values themselves. This has always been done merely as a convenience in calculation, since the quantities resulting are easier to handle. In all slope-deflection solutions made in recent years in which the number of unknowns is large, this method of "balancing angles" or "converging end slope values" has been used, but the convergence or balancing has started from the zero end angle case in which M_F is the fixed-end bending moment.

(3) In Table 1(*b*) a solution is given for the case of elastic angle slip at the joints, which has the advantage of making a trial solution unnecessary.

[13] For an example, see "Statically Indeterminate Stresses", by John I. Parcel and G. A. Maney, Members, Am. Soc. C. E., Fig. 142, opposite p. 276, John Wiley and Sons, 1936.

Note the close check on values of final moments. In another problem checked by the writer (see Table 2), the case of both elastic and plastic joints is treated. The "corrections due to yielding angle" (see Table 2(*b*) where this correction is -127) herein used can only be,

$$\Delta M_{BC} = -2 E K_{BC} [2 \phi_{BC} + \phi_{CB}] \ldots\ldots\ldots\ldots\ldots (9)$$

using the slope-deflection formula in which ϕ-values are angular discontinuities. When they are elastic, ϕ_{BC} becomes $\frac{M_{BC}}{R_{BC}}$ and $\phi_{CB} = \frac{M_{CB}}{R_{CB}}$ (in which the R-values are the moduli of rotation) which are ratios of bending moment to corresponding angular rotation of the end of the member, in radians. In Table 2(*b*), this correction becomes $-2 E K \left(\frac{2 M_{BC}}{R_{BC}}\right)$ for M_{BC}, or $-0.498 M_{BC}$.

(4) Particular attention is called to the case of "plastic" discontinuity of end angles as included in the author's problem in Fig. 10 and in the writer's corresponding solution in Table 2(*b*). The percentage change in end moments due to lack of joint rigidity is listed in Column (9), Table 2(*b*). This percentage varies from a decrease of 41% to an increase of 7%, and emphasizes the need for a solution as given in Table 1(*b*) which eliminates guesswork regarding the final moment values.

(5) If there is a "plastic" feature of the angular discontinuity, why should not all end moments closely approach zero finally? A plastic condition *per se* means "give" under continued load with a corresponding steady increase of ϕ-values with time. If this simple beam condition is approached, why indulge in rigid-frame analysis for frames with "flowing" or "plastic" joints?

Tables 1 and 2 may be clarified by examining Table 1(*a*), as follows:

(*a*) Line 2 indicates that $-8\,\theta_B - 4\,\theta_D + 8.33 = M_{BD}$, which is a specific statement of the general equation applicable to this case. The general form would be stated:

$$M_{BD} = \pm M F_{BD} - K_{BD} (2\,\theta_B + \theta_D) \ldots\ldots\ldots\ldots\ldots (10)$$

in which, $\pm M F_{BD} = +8.33$ ft-kips; $2 K_{BD} = 8$; $K_{BD} = 4$; and θ_B and θ_D are multiples of the angular rotation of Joints B and D when equilibrium is reached.

(*b*) Line 8 shows that $\Sigma M_B = 0$; or that the net rotational tendency of all moments acting on Joint B is zero when equilibrium is reached. The expressions for M_{BA} in Line 1 and for M_{BD} in Line 2 are added to obtain the joint equation of Line 8, which is: $11\,\theta_B + 4\,\theta_D = +8.33$. Likewise, equations of Lines 3, 4, 5, and 6, are added to obtain the equation (Line 9) for Joint D.

(*c*) Lines 10, 11, and 12, indicate that $1.000\,\theta_B + 0.364\,\theta_D = 0.758$ and that $0.186\,\theta_B + 1.000\,\theta_D = -0.566$. Lines 13, 14, 15, and 16 are the balancing steps for the angle values ($0.364\,\theta_D$ for θ_B, and $0.186\,\theta_B$ for θ_D) at the far end, converging until the corrections approach zero.

(*d*) Line 17 represents final values of $2 E \theta$, or additions of five preceding quantities, which are substituted in the original moment expressions (Lines 1 to 7, inclusive) and values such as − 3.00 ft-kips are obtained for all member ends, such as End B of Member BA (M_{BA}). It will be noted that the sum of all moments around a joint must be equal to zero, a desirable check on the numerical accuracy of the work.

PAUL ANDERSEN,[14] ASSOC. M. AM. SOC. C. E. (by letter).[14a]—In many respects Professor Grinter's method will prove a more flexible tool to the designing engineer than the Cross method. It is true that the latter has the advantage over the former in that it makes use of moments throughout its procedure and leads directly to the actual bending moments; but this disadvantage of the Grinter method is more than compensated for by the facts that it is self-checking and that the actual work involved in establishing continuity has been reduced materially. Thus, in using the Cross method, it is necessary to balance the moments at each end of a member, whereas by balancing angle changes, only one set of values is needed at each joint.

The procedure used in solving the example in Fig. 2 seems incorrect; Joint B is balanced and the carry-over angle from B is added to the angle changes of Joint D and included in the first process of balancing Joint D. The proper procedure is to balance all joints first and then to carry over the angle changes, and repeat.

Side-Sway.—Instead of obtaining the corrections for side-sway by imposing equal angle changes at the top and bottom of each column, the writer suggests applying, at the column tops, any set of angle changes inversely proportional to the column heights, and distributing these changes throughout the structure. It is readily seen that, by equating horizontal deflections before and after balancing the angle changes, the moment at the top of a column is:

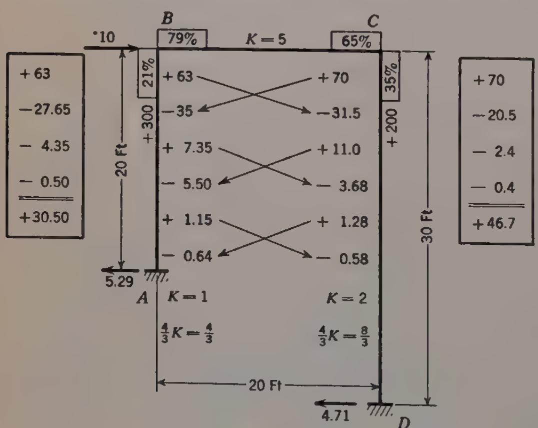

FIG. 14.

$$M = \frac{Vh}{2} \times \frac{\phi - \theta}{\phi - \frac{3}{4}\theta} \quad (11)$$

in which ϕ = an arbitrary angle change of the top of the column; θ = the resulting joint rotation; V = column shear, proportional to $\frac{K}{h^2}$; and h = column height.

This procedure is applied to the two-legged bent

[14] Balboa Heights, Canal Zone.

[14a] Received by the Secretary, October 26, 1936.

shown in Fig. 14 in which angle changes of 300 and 200 are applied to the column tops, resulting in joint rotations of 30.5 and 46.7, respectively, after continuity has been established. The final moments (see Fig. 14) are,

$$M_B = \frac{1}{2} \times 5.29 \times 20 \times \frac{300 - 30.5}{300 - \frac{3}{4}\,30.5} = 51.4 \text{ ft-kips}$$

and,

$$M_C = \frac{1}{2} \times 4.71 \times 30 \times \frac{200 - 46.7}{200 - \frac{3}{4}\,46.7} = 65.6 \text{ ft-kips}$$

WILLIAM F. LUCE,[15] JUN. AM. SOC. C. E. (by letter).[15a]—A valuable tool for the analysis of continuous frames is contained in this paper. Computations by the method introduced involve only simple arithmetic with no simultaneous equations, and can be made, in many cases, in a fraction of the time required by the so-called "exact" methods, such as the slope-deflection method, the method of least work, etc. Furthermore, both the development and the application of the method involve concepts familiar to the structural engineer.

The presentation by Professor Grinter has been so complete that further explanation is scarcely necessary. The following development, however, may help some readers to visualize the balancing process and the determination of the joint rotation factor. In Fig. 15(a), the members, A, B, C, and D, have stiffness ratios $\left(\frac{I}{L} = K\right)$ of K_A, K_B, K_C, and K_D, respectively.

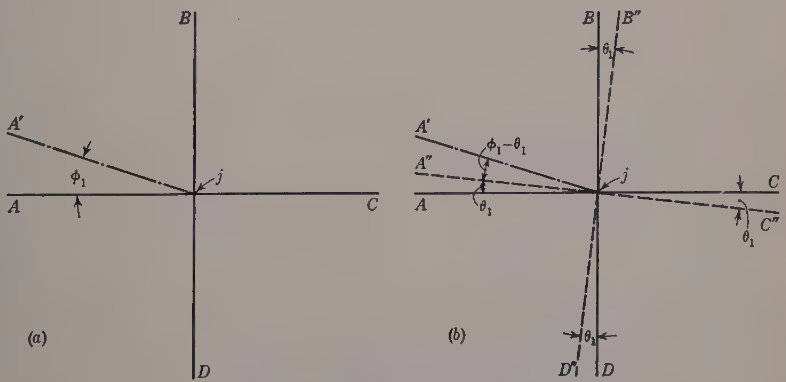

FIG. 15.—BALANCING A JOINT.

If Member A is rotated clockwise about Point j through an angle, ϕ_1, to the position, A', the joint is discontinuous. In restoring continuity, by moments applied to the members, at Point j, Members B, C, and D will be rotated clockwise through an angle, θ_1, to the new positions, B'', C'', and D'', whereas

[15] Univ. Fellow in Structural Eng., Univ. of Michigan, Ann Arbor, Mich.

[15a] Received by the Secretary November 6, 1936.

Member A will be twisted counter-clockwise through an angle $(\phi_1 - \theta_1)$ to the position, A''. The internal moments balance at Joint j:

$$M_A = M_B + M_C + M_D \qquad (12)$$

and, since the moment in a member depends upon the product of its stiffness and the angle through which it rotates:

$$(\phi_1 - \theta_1)\ K_A = \theta_1\ K_B + \theta_1\ K_C + \theta_1\ K_D \qquad (13)$$

or,

$$\theta_1 = \phi_1 \frac{K_A}{K_A + K_B + K_C + K_D} = \phi_1 \frac{K_A}{\Sigma K} \qquad (14)$$

Factor $\frac{K_A}{\Sigma K}$ is termed the joint rotation factor for Member A at Joint j. The development is equally simple if two or more members are given an original angular discontinuity.

It is noted from the paper that members having their ends free to rotate are the norm in this method, and that the stiffness values for fixed-end members are corrected by the factor, $\frac{4}{3}$. This is the reverse of the procedure used in the method of balancing moments.

The writer has solved problems on secondary stresses in trusses, wind-stress analysis of building frames, and joint slip, by the method of balancing angle changes. The structure investigated for secondary stresses was a symmetrical Pratt truss of eight panels. Angle changes were computed by Dean Ketchum's formulas as suggested by the author. The balancing required four to five cycles per joint. The end moments found in this manner balanced within 1% at every joint, providing a check on the work. As a final check, the secondary stresses were calculated by the method of balancing moments, using deflections taken from a Williot-Mohr diagram. The variation of stress values as obtained by the two methods was small in most members, and the maximum difference was 400 lb per sq in. It was observed, however, that the balancing process in either case should be started with comparatively large numbers if the final results are to be accurate.

The first wind-stress problem investigated was that of a 6-story building[16], the analysis being made by estimating the shape of the deflected structure as outlined by the author elsewhere[4]. The final moments obtained, requiring only one balancing process, checked those calculated by balancing moments within 1% to 2 per cent. The latter method, however, seems to be definitely better suited to the solution of wind-stress problems. Only four cycles of balancing moments were required to give the same accuracy as six or seven cycles of balancing angle changes, and, of course, did not require the use of the slope-deflection equations at the end of the process. The analysis

[16] Structural Engineer's Handbook, by Milo S. Ketchum, Hon. M. Am. Soc. C. E., 1924 Edition, p. 102.

[4] "Wind Stress Analysis Simplified", by L. E. Grinter, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 610.

of the lower stories of another building frame[17], also indicated that the value of the check embodied in the method of balancing angle changes is decreased because of the introduction of different criterion ratios in adjacent stories. The method of balancing moments has the further advantage in the analysis of frames in which joint translations occur, such as wind-stressed buildings, that fixed-end moments may be added during the balancing process to offset joint restraints. A similar procedure for the method of balancing angle changes is not apparent.

The solution of joint slip problems and the plotting of influence lines for continuous beams can be performed very successfully by balancing angle changes.

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[17] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 664.

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DISCUSSIONS

SELECTION OF MATERIALS FOR ROLLED-FILL EARTH DAMS

Discussion

By Messrs. T. T. Knappen, and Paul Baumann

T. T. Knappen,[15] M. Am. Soc. C. E. (by letter).[15a]—There are certain fundamental conceptions essential to the design of a dam. At a particular site an engineer may consider some form of concrete structure or a structure built of earthen materials, which may include rock. In considering an earth dam, the engineer has a choice between dry and hydraulic methods of construction. For the dry method practice has standardized on the placement of material in thin layers, with suitable compaction by rollers of various types, or by power-tamping or vibrating equipment. For the economical design of an earth dam, it is essential, first, to utilize all materials from required excavations; and, second, to construct the remainder of the dam from the materials available in the vicinity. The greatest economy generally results from the use of materials closest to the site.

In this paper, the author has based his selection to a large extent on grain-size distribution. This is a logical method in the preliminary stages. In Figs. 6 and 7, he has given the proposed limits for materials suitable for impervious sections of rolled-fill earth dams. The writer agrees that, in general, material falling within these limits will be satisfactory. However, it is entirely possible to use materials that do not come within these limits. First, considering materials that are finer than the upper curves on these two diagrams, although such materials may be somewhat difficult to handle and may be structurally somewhat unstable, they may furnish, nevertheless, an excellent impervious core for a dam embankment. In the computation for such cases the outer shell must be depended upon for structural strength. The dam may be analyzed by a method recently outlined by the writer.[16]

Considering the two lower curves which represent the limit of permeability, it is perfectly possible to use materials of greater permeability than those indicated. It may be conceded that an earth dam can be safely designed

Note.—The paper by Charles H. Lee, M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[15] Prin. Engr., Chf., Flood Control Section, North Atlantic Div., U. S. War Dept., New York, N. Y.

[15a] Received by the Secretary September 25, 1936.

[16] "Calculation of the Stability of Earth Dams", presented before the World Power Conference, Washington, D. C., September, 1936.

and built, provided it is properly designed, with the expectation of a large leakage. The approximate upper limit, indicated by the author, of 1 gal per sq ft per day, with an hydraulic gradient of 1 is extremely conservative. This is equivalent to a value of k in Darcy's formula,

$$Q = k\,i\,A\,t \qquad (2)$$

of 0.4×10^{-4} cm per sec. In the case illustrated, the author deduced that the leakage would be 0.004 cu ft per sec with a coefficient of 0.1 gal per sq ft per day. From a structural standpoint, a leakage of a thousand times this amount is not necessarily excessive with proper design. In other words, a permeability coefficient of 100 gal per sq ft per day might be satisfactory in centimeter-gram-second units (40×10^{-4} cm per sec.) It would seem to the writer that the upper limit of permissible seepage is largely a matter of economics. The cost of reducing the leakage should be balanced against the value of the water saved. In the case of flood-control dams, or dams at which constant releases are required, it may be of no economic consideration whatever. In such cases, it becomes merely a structural problem of designing the embankment to pass the leakage safely. Recently, the writer had occasion to design a dam for which the estimated leakage through the foundations, with water at spillway level, amounted to 50 cu ft per sec. In such a case, it would be unwarranted to go to great expense to obtain an impermeable embankment.

The writer cannot agree with the author in eliminating the siltstone (Sample *L*15–*CG*, Table 2) for use in the Coyote Dam on the basis of the information set forth in the paper. It would appear that this material could have been used advantageously at least in the center one-fourth of the embankment, with resulting economy. Certainly, more exhaustive tests to determine its structural nature in terms of cohesion and angle of internal friction under saturated conditions in the embankment should have been made before discarding its use in the center section of the dam, and there is a possibility that it might have been used safely for a large percentage of the embankment.

In the paragraph following Fig. 4 the author makes the statement that "the data in Figs. 3 and 4 indicate that for permanent stability of compacted earth material the clay fraction should not exceed 30 to 35%; nor should it be less than 3 to 15%, the latter limits depending on the degree to which the material is graded." Based on this statement, the engineer having such a material as Sample *L*15–*CG* would be precluded from constructing a dam, whereas it can be used with proper design. A combination of Sample *L*15–*CG*, or even Sample *L*8–*A* for a core and Sample *L*15–*S*10 (or, preferably, a coarser material) for the shells will result in an excellent and economical dam, possibly a better one than that built uniformly with Sample *L*15–*C*9.

There are few materials that cannot be used successfully, to some extent, in the construction of earth dams. The materials approved by the author may be used with little additional investigation, but for true economy in difficult designs every analytical resource of the science of soil mechanics should be called upon to produce a design utilizing the closest available materials to produce a safe and enduring structure. The writer knows of more than one dam that could never have been built had the author's limitations been applied.

PAUL BAUMANN,[17] M. AM. SOC. C. E. (by letter).[17a]—Due to the rapid exhaustion of suitable dam sites it is believed that, in the future, rolled-fill dams will gradually exceed masonry dams in number. Hence, this paper is particularly timely and valuable.

The author points out five principal requirements for suitability of material for the impervious part of a rolled-fill earth dam. He further stresses the importance of grading the material to satisfy these requirements. This tends to imply that unless material which conforms to the specifications of grada-

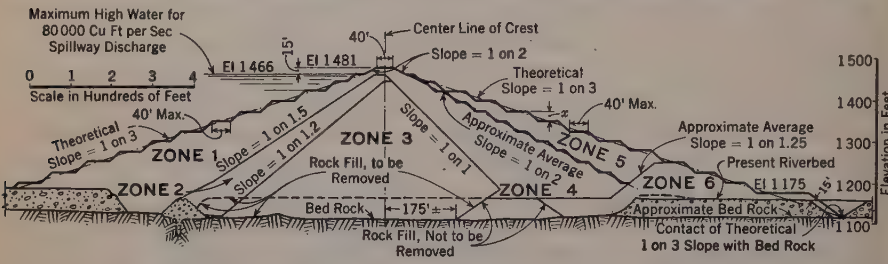

FIG. 8.—COMPOSITE MAXIMUM SECTION, SAN GABRIEL DAM NO. 1.

tion is available in the vicinity of a site chosen for a rolled-fill dam, such a dam cannot be built. The writer's experience, acquired during the construction of San Gabriel Dam No. 1, does not substantiate this contention. Material that is not initially suitable for rolled-fills, because of gradation defects, can be made suitable by proper compaction.

Two widely different types of materials have been placed and compacted in San Gabriel Dam No. 1: A clayey sand or loam in Zone 2; and quarry-run

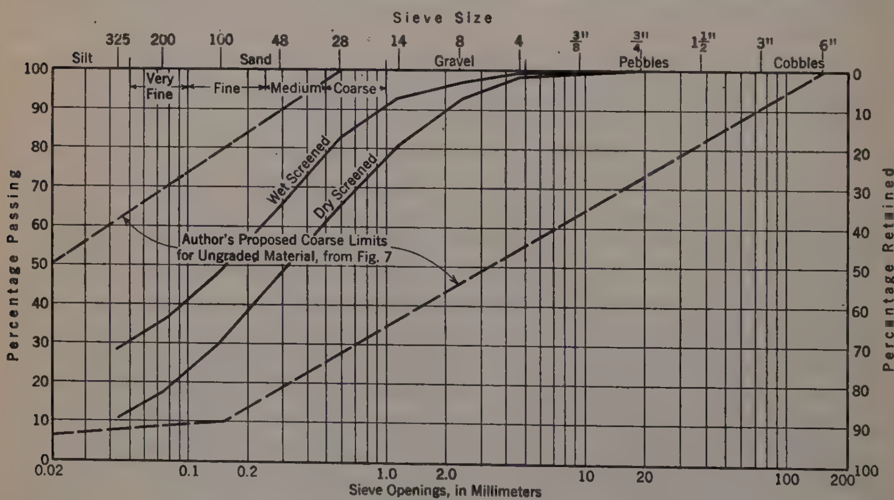

FIG. 9.—GRADATION OF SOIL COMPACTED IN ZONE 2.

material which has passed through a 6 by 9-in. grill in Zone 3. The location of these zones is shown in Fig. 8 which is composite cross-section of the dam.

[17] Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

[17a] Received by the Secretary November 10, 1936.

In order to illustrate the comparative initial and final gradation, a typical screen analysis is shown for each of the two materials in Figs. 9 and 10. Whereas, the fineness modulus of the wet clayey sand is 1.17 for the average sample before compaction and is not changed materially through compaction,

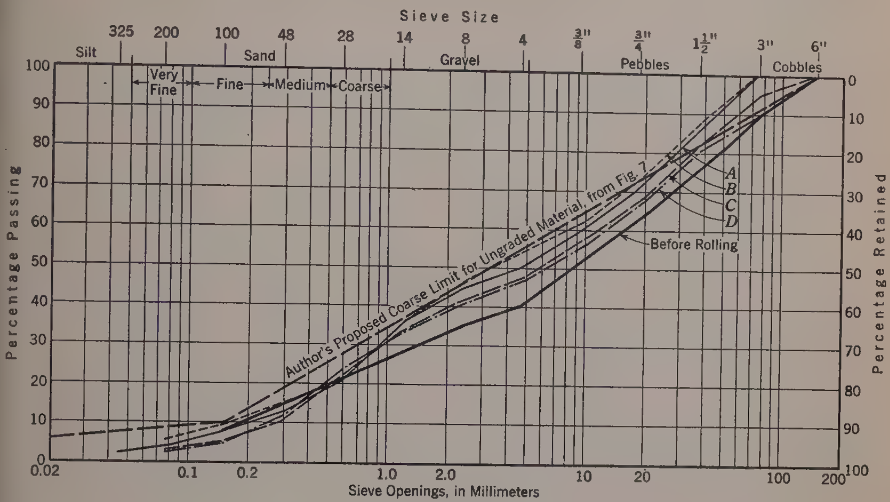

FIG. 10.—EFFECT OF ROLLING (TEN TRIPS WITH A 12.1-TON ROLLER) ON GRADATION OF MATERIAL PASSING A 6 BY 9-FOOT GRILL.

the average fineness modulus of the quarry-run material is 5.72 before compaction and 5.06 after compaction; and, although the material in Zone 2 conforms approximately to what might be called standard specifications for impervious material for a rolled-fill dam, the material used in Zone 3, (including rock as large as 9 in.) would certainly not appear to satisfy all five requirements, at least on a first examination.

As a matter of fact, with the rolling equipment used at San Gabriel Dam No. 1, which was specially designed for the job, material in Zone 3 requires less effort for compaction than material in Zone 2; that is, whereas twelve trips are required to produce the specified compaction in Zone 3, between sixteen and twenty trips are required to produce such compaction in Zone 2. The specifications for Zone 3 provide that the dry density of the fines (that is, particles 0.25 in., or smaller) must be compacted so as to have a dry density of not less than 120 lb per cu ft., whereas the specifications for Zone 2 provide a compaction which shall produce a dry density of the composite material of not less than 115 lb per cu ft.

The reason for eliminating all particles greater than 0.25 in. from consideration with Zone 3 material is that the fines filling the voids between the coarse material, or rock, actually govern the physical behavior of the composite compacted material. The division between coarse and fine of the 0.25-in. size was adopted from standard practice pertaining to concrete aggregates, and has been found through experimentation to be justified.

Although the shearing strength of the saturated material in Zone 2 is less than that of the saturated fines in the material of Zone 3, the relative permeability is reversed; that is, material in Zone 2 is less permeable than the fines in the material in Zone 3 at the specified densities. The average percolation rate under a gradient of 1 and approximately 60° F for material in Zone 2 at the specified density is 0.5 ft per yr, whereas for the fines in the material of Zone 3 under the specified density, it averages 1.5 ft per yr.

The stability under saturation of any fill material is governed primarily by its dry density. In other words, the internal friction or shearing strength which governs the stability of the material depends on the dry density. Shearing tests on saturated fines of the material in Zone 3 have shown shearing strengths varying from 22 lb per sq in. for a vertical load of 25 lb per sq in., to a shearing strength of 525 lb per sq in. for a vertical load of 1 200 lb per sq in. Due to this governing factor (that is, the dry density of the material), it is quite obvious that the conditions for stability of a hydraulic-fill dam under saturation and the conditions for stability of a rolled-fill dam under saturation are necessarily at variance. With the hydraulic-fill process it is not possible to create an initial dry density of the material in excess of a relatively low limit, whereas with the rolled-fill, or mechanically compacted-fill process, the attainable dry density is approaching that of the solid material. The composite fill in Zone 3, for example, in place, has an average weight of 150 lb per cu ft., which indicates that it is approaching the physical state of an artificial stone. The specific gravity of the material used in Zone 3 averages 2.78, which means a solid weight of 173.5 lb per cu ft. The average dry weight of the composite material in place was found to be 143 lb which would mean that the porosity of the composite fill in Zone 3 is approximately 17.5 per cent. Typical results of density tests are shown in Table 6.

TABLE 6.—SOIL CHARACTERISTICS SUPPORTING THE CURVES IN FIG. 12

Sample taken from Zone: (1)	FIELD CONDITIONS: Wet weight, in place, in pounds per cubic foot (2)	FIELD CONDITIONS: Dry weight, in pounds per cubic foot (3)	FIELD CONDITIONS: Percentage of moisture in fines (4)	FIELD CONDITIONS: Percentage of rock (5)	FIELD CONDITIONS: Number of trips by a 12.1-ton roller (6)	COMPACTION DATA: Specific gravity (7)	COMPACTION DATA: Percentage passing a No. 200 sieve (8)
3.........	151.5	128.7*	9.6	42.4	12	2.78	25.4
2.........	133.9	118.2†	13.3	3.2	20	2.79	36.3

* Fines. † Composite.

The reason for the small porosity and, consequently, the small permeability of material in Zone 3, is the breakdown that occurs during the rolling of the material as is evident from Fig. 10. The effect of compaction relative to breakdown for variable moisture is further illustrated in Fig. 11. Fig. 11(*a*) shows the increase in breakdown due to twenty-five hard tamps with a 5.5-lb tamper over the breakdown due to five 4-in. drops. The maximum, additional breakdown coincides with the peak of compaction.

Furthermore, the initial compaction of granular material and the breaking strength of the particles are governing factors in regard to its stability under saturation, because the water which, in a saturated condition, completely fills the voids, cannot escape in case of sudden compression of the material, but will, itself, be put under stress. Hence, if the compression is of sufficient magnitude to crush the parts in contact between particles, the shearing strength may readily drop to a critical value due to load being transmitted through water on the one hand and to the production of very fine particles or flour caused by the crushing on the other. These fine particles, after crushing, will naturally mix with the water and will form slime which tends to lubricate the contact surfaces and minimize internal friction.

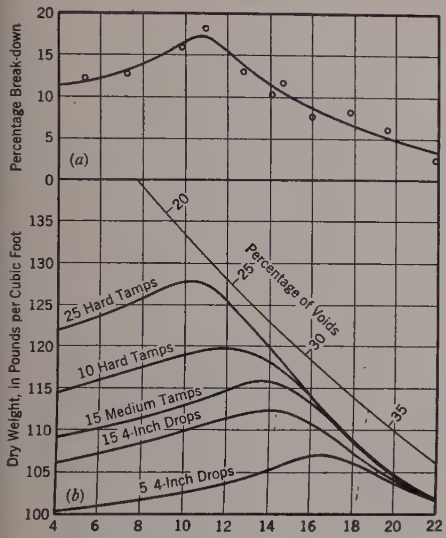

FIG. 11.—COMPARISON OF EFFECT OF MOISTURE CONTENT ON COMPACTION AND ON BREAKDOWN.

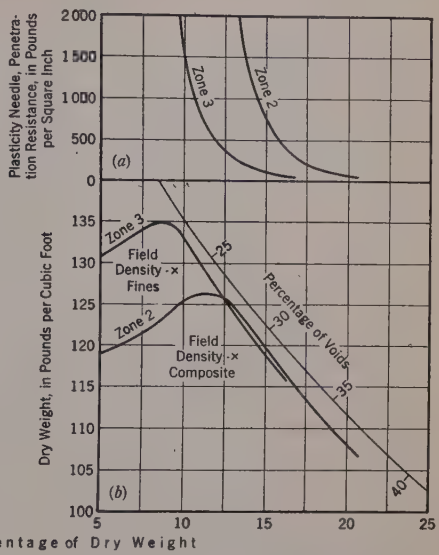

FIG. 12.—SOIL CHARACTERISTICS.

In regard to cohesion due to the surface tension of water which is always present in relatively fine material with one or more free surfaces exposed to air, such cohesion cannot develop under saturation. This is the principal cause of the softening up of granular soils, as well as clays, under saturation. Hence, all tests bearing on the stability of a relatively impervious fill should be conducted on saturated material in order to determine the safety of such a structure in its critical condition.

The author does not mention the penetration needle in connection with soil analysis and gives no reason for this omission. Based on tests (as yet incomplete) conducted at the laboratory at San Gabriel Dam No. 1, it is believed that the significance of the penetration or plasticity needle has been misinterpreted in the past. These tests tend to show that the penetration needle, although an excellent moisture indicator on the wet side of the peak

of compaction (see Fig. 12) is not reliable as an indicator of stability of a fill if the pore water is under stress.

The safety of a dam fill composed of insoluble material depends directly on the shearing strength of its grain structure as long as over-topping is prevented. The shearing strength for a given density, in turn, depends on the rate of percolation and the pore-water stress. Although the rate of percolation primarily depends on the grain size, the pore-water stress depends on the condition of loading and particularly the rate at which load is being applied. Hence, the influence of the sudden application of load on a saturated dam fill, due to earthquake, for example, is an important part of the analysis of safety.

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DISCUSSIONS

STRUCTURAL APPLICATION OF STEEL AND LIGHT-WEIGHT ALLOYS A SYMPOSIUM

Discussion

By Messrs. E. Mirabelli, R. W. Vose, Raymond H. Hobrock, William F. Clapp, J. C. Hunsaker, Horace C. Knerr, and F. T. Sisco.

E. Mirabelli,[103] M. Am. Soc. C. E. (by letter).[103a]—In discussing safety factors and working stress, Mr. Karpov points out that the safety of a design in any redundant construction may be influenced by the fact that weak members can transfer part of their burden to other parts of the structure. In this manner the actual factor of safety is raised above that indicated by a comparison of computed stress with yield point or ultimate strength of material. In a simple beam there occurs a similar transfer of load from over-stressed outer fibers to under-stressed interior fibers, and the calculated stress does not give an exact indication of the margin available to the useful load-carrying capacity of the beam. After the maximum fiber stress has reached the yield point a certain amount of additional load may be applied without causing distress and, within definite limits, the beam deflection will continue nearly in direct proportion to the loading. Ultimately, the useful limit of the beam is reached, and deflection will increase rapidly with little or no increase in loading. The useful limit is affected to a large extent by the type of beam section and also to a small degree by the type of loading. These facts have been known for some time from results of tests and are used to some extent in the structural analysis of airplanes[104]. A process which would lead to a quantitative prediction of the behavior described herein would, to some degree, attain Mr. Karpov's apparent

Note.—This Symposium was presented at the meeting of the Structural Division at Pittsburgh, Pa., October 14–15, 1936, and published in October, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the Symposium.

[103] Asst. Prof. of Structural Design, Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[103a] Received by the Secretary, October 31, 1936.

[104] "Airplane Structures", by Niles and Newell, John Wiley and Sons, N. Y., 1929, p. 239.

aim which is to diminish the gap between predicted and actual behavior. Such a process for beams of ordinary carbon steel may be based on an idealization of the stress-strain diagram into a series of straight lines, as shown in Fig. 46(*a*). The strain-hardening region to the right of $n_1 \epsilon$ is

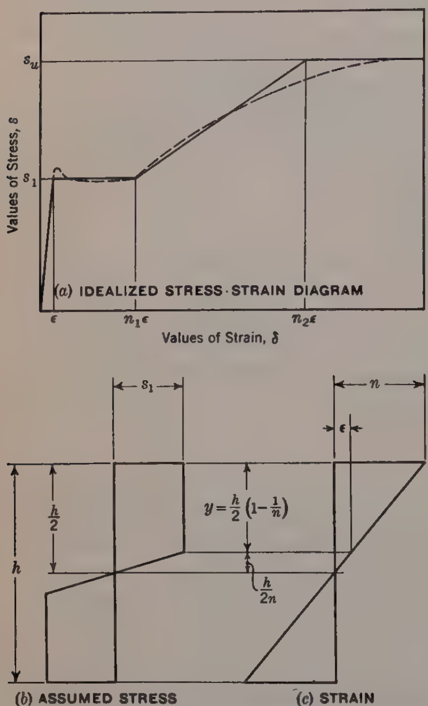

FIG. 46.—STRESS AND STRAIN DIAGRAMS.

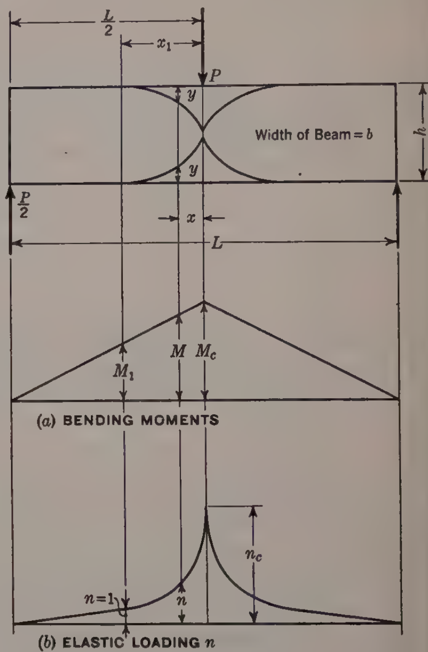

FIG. 47.—RECTANGULAR BEAM WITH A CONCENTRATED LOAD AT THE CENTER.

unimportant because failure takes place in a form of a rapid increase in the rate of vertical deflection before the outer fibers are strained sufficiently to reach this region. (This discussion refers to plastic failure only and not to elastic failure due to lack of lateral support.) The assumptions are made that through the entire range of stress, including the plastic range, a plane section before bending remains plane after bending[105] and also that the stress-strain relation is the same for tension and compression[106]. The stress distribution over a symmetrical beam section then will be as shown in Fig. 46(*b*), with a portion near the neutral axis in a state of elastic stress and the remainder of the section in a plastic state.

To illustrate the method of developing the deflection equations, the rectangular beam shown in Fig. 47 will be considered with a single concentrated load at mid-span of sufficient magnitude to produce a region of

[105] "Elastizität und Festigkeit", by Bach-Bauman, Ninth Edition, Berlin, 1924, p. 289.

[106] "The Strength of I-Beams in Flexure", by H. F. Moore, *Bulletin No. 68*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

plastic stress. The limits of the plastic regions are defined horizontally by x and vertically by y. Using Fig. 46(b) and Fig. 46(c) and taking moments about the neutral axis, it can be shown that the moment of resistance is,

$$M = \frac{1}{6} s_1 b h^2 \left(\frac{3}{2} - \frac{1}{2 n^2}\right) \qquad (14)$$

Now, let P_1 represent the smallest load that would cause a maximum bending stress equal to the yield point stress, and let M_1 represent the corresponding maximum bending moment. Then, from the equality of external and internal moments it is obvious that,

$$M_1 = \frac{1}{4} P_1 L = \frac{1}{6} s_1 b h^2 \qquad (15)$$

At the section where $x = x_1$ (see Fig. 47), the bending moment is,

$$\frac{P}{2}\left(\frac{L}{2} - x_1\right) = \frac{1}{6} s_1 b h^2 = \frac{1}{4} P_1 L \qquad (16)$$

from which,

$$x_1 = \frac{L}{2}\left(1 - \frac{P_1}{P}\right) \qquad (17)$$

Equation (17) defines the horizontal limit of the plastic region along the outer fibers.

The strain along the outer fibers at any section which cuts the plastic region is measured by the value of the quantity, n, at that section. To obtain an expression for the strain factor, n, substitute Equation (15) in Equation (14) and equate internal and external moments, thus:

$$\frac{P}{2}\left(\frac{L}{2} - x\right) = \frac{1}{4} P_1 L \left(\frac{3}{2} - \frac{1}{2 n^2}\right) \qquad (18)$$

When solved for n, Equation (18) gives,

$$n = \sqrt{\frac{1}{3 - 2\left(\frac{P}{P_1}\right)\left(1 - 2\frac{x}{L}\right)}} \qquad (19)$$

When $x = x_1$, it is obvious that $n = 1$, and, since in the purely elastic region the strain is proportional to the bending moment, it follows that in the region, $\frac{L}{2} > x > x_1$, the strain factor is given by the expression,

$$n = \frac{L - 2x}{L - 2x_1} \qquad (20)$$

The strain factor, n, may be used now to obtain deflections from the relation,

$$\delta = \int m \, d\theta \qquad (21)$$

in which m is the bending moment at any section due to a unit load applied at the section for which the deflection, δ, is desired, and $d\theta$ is the angular distortion in any element, dx. From Fig. 46(c) it is apparent that the angular distortion is,

$$d\theta = \frac{2\,n\,\epsilon}{h}\,dx \qquad \text{.......................(22)}$$

If the deflection at mid-span is to be determined,

$$m = \frac{1}{2}\left(\frac{L}{2} - x\right) \qquad \text{.......................(23)}$$

Substituting Equations (22) and (23) in Equation (21) and then performing the integration with the help of Equations (17), (19), and (20), and simplifying:

$$\delta_c = \frac{\epsilon\,L^2}{h}\left\{\left(\frac{P_1}{P}\right)^2\left[\frac{5}{6} - \left(\frac{3}{4} - \frac{1}{12\,n^2_c}\right)\frac{1}{n_c}\right]\right\} \qquad \text{........(24)}$$

in which n_c is the strain factor at mid-span and may be calculated by use of Equation (19). If the quantity within the braces is given a substitution value, u, Equation (24) may be simplified thus:

$$\delta_c = \frac{u\,\epsilon\,L^2}{h} \qquad \text{.........................(25)}$$

An alternative method for calculating deflections is by using quantities, $d\theta$, as "elastic weights." From Equation (22), it is seen that for a beam of uniform height the only variable is the quantity, n. This quantity may be used alone as an "elastic loading" if the results are corrected by multiplying by the constant, $2\frac{\epsilon}{h}$. The deflection at any point of the beam is the "bending moment" resulting from the application of the "elastic loading."

The foregoing method may be extended to beams with other types of cross-section. The loading-deflection curves shown in Fig. 48 were plotted from calculated deflections. The I-sections were assumed to have a ratio of flange thickness to depth of beam equal to 0.05 and a ratio of web thickness to width of flange equal to 0.10. These are average values for standard I-beams. The diamond-shaped section is a square with loading in the plane of a diagonal. In all calculations it was assumed that $n_1 = 12$ (see Fig. 46(a)) and that the slopes of the two inclined lines of the idealized diagram are in the ratio, 1:80. These are average values for medium carbon steel.

From the definition of P_1, it follows that the ordinate, $\frac{P}{P_1} = 1.0$ (Fig. 48) shows when yield point stress begins in the most stressed material. These curves indicate a yield point in flexure which differs from the yield point in direct stress and which varies widely with the shape of the beam section.

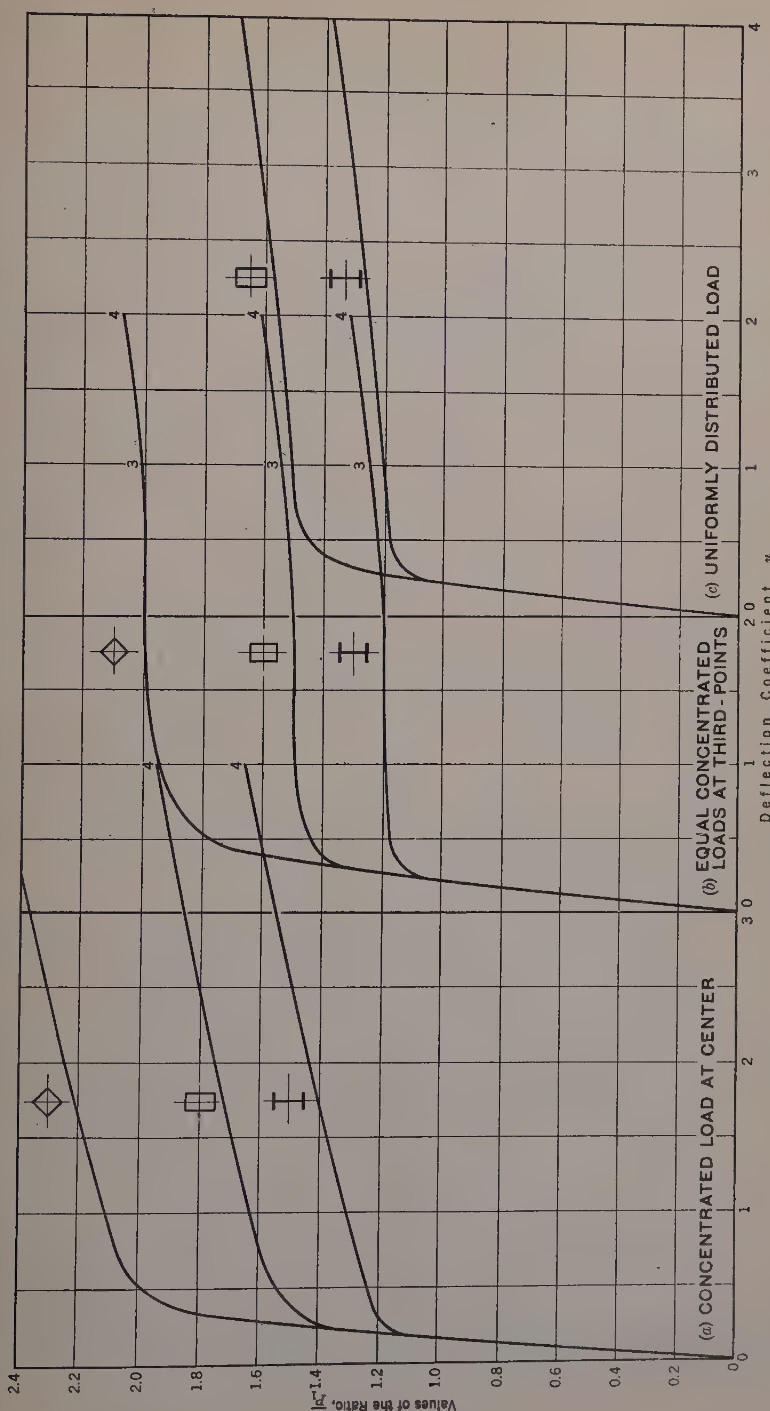

FIG. 48.—LOAD DEFLECTION CURVES (SEE EQUATIONS (24) AND (25)).

These curves also indicate a possible exception to Mr. Karpov's statement that the stress-strain curves of Fig. 2 demonstrate the non-applicability of Hooke's law to low-strength steel stressed above the yield point. If Hooke's law may be interpreted as meaning that deflections are proportional to the forces which produce them, it is evident that sometimes the law is applicable with close approximation even when the steel is stressed above the yield point because the deflections continue very nearly in proportion to the loading for a considerable distance. The calculated curves are in good agreement with the published results of a number of experiments[107]. If the yield point in flexure is to be taken as the limit of the useful carrying capacity of a beam (that is, the point on which the factor of safety is to be based), it is evident that such a factor will vary with the shape of the beam. For example, using a working stress of 18 kips per sq in. for steel having a yield point of 36 kips per sq in., the factor of safety in flexure for the square section on edge is 4, for the rectangular section, 3, and for the typical **I**-section, 2.4. To obtain a uniform safety factor of 2.4, for example, the working stresses should be 30 kips per sq in. for the square section on edge, 22 500 kips per sq in. for the rectangular section, and 18 kips per sq in. for the typical **I**-section.

If the maximum bending stresses as computed by the usual beam formula should accidentally exceed the yield point of the material by a small amount, there would be an apparent over-stress, but no sudden yielding of the beam. A small permanent deformation of the outer fibers may occur but the curves indicate that for the beam as a whole there could be no appreciable residual deflection on removal of the load. Residual stresses of tension might remain in the top fibers and compression in the lower fibers. However, this condition is not worse than that in which beams or plates are straightened or bent cold in fabricating shops. It must be assumed in the foregoing discussion that the range of stress is not sufficient to cause the fatigue failure, which Mr. Karpov has emphasized.

There is no intention of suggesting the use of design stresses as high as the yield point of the material. It is simply indicated that an occasional loading to such a point or somewhat beyond produces no permanent harmful effect.

There is still a question as to the effect of plastic flow or creep whereby long-continued, excessive loading may cause progressive increase in deflection. It appears reasonable to believe that plastic flow of an appreciable magnitude occurs only if the plastic region extends over a considerable part of the cross-section of the beam. Otherwise, the resistance of the elastically stressed portion of the section prevents further deformation.

[107] Test data may be found in the following publications: "Experiments on the Yield Point of Steel under Transverse Tests", by Sir A. B. W. Kennedy, *Engineering*, London, June, 1923, Vol. 115, p. 736: "The Strength of I-Beams in Flexure", by H. F. Moore, *Bulletin No. 68*, Eng. Experiment Station, Univ. of Illinois; "Strength of Light I-Beams", by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., and J. O. Draffin, M. Am. Soc. C. E., *Bulletin No. 241*, Eng. Experiment Station, Univ. of Illinois; "Beitrag zur Frage der tatsachlichen Tragfähigkeit einfacher und durchlaufender Balkenträger", von Maier-Leibnitz, *Die Bautechnik*, Berlin, 1928, Vol. 6, pp. 11, 27; and "Versuche mit eingespannten und einfachen Balken von I-Form", von Maier-Leibnitz, *Die Bautechnik*, Berlin, 1929, Vol. 7, p. 313.

The yield point in flexure for I-beams varies with the proportions of flanges and webs. The curves of Fig. 49 show the effect of variations in such proportions. It is interesting to note that keeping the flange thickness

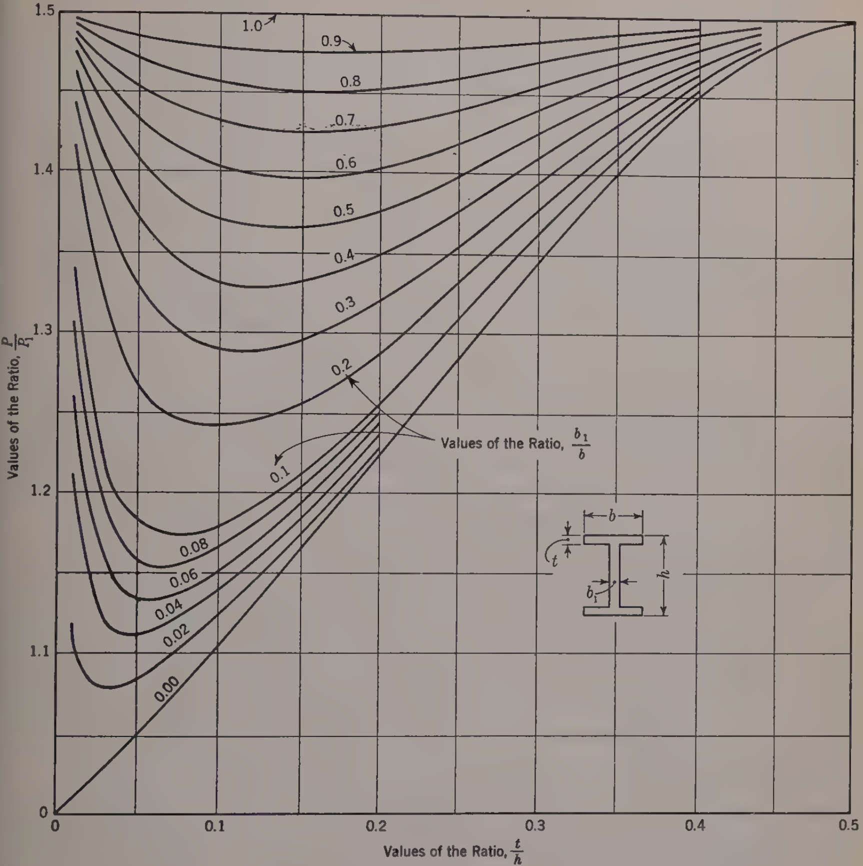

FIG. 49.—LOADS AT THE YIELD POINT FOR AN I-SECTION.

constant and increasing the web thickness always results in an increased ultimate strength, whereas keeping the web thickness constant and increasing the flange thickness does not always result in an increased ultimate strength.

The foregoing methods for simple beams may be applied to continuous beams with interesting results. For example, for the rectangular beam shown in Fig. 50, P_1 is one of a pair of equal loads which when applied at mid-span will start yield-point stress at a section over the middle support, and P is the corresponding load which will start yield-point stress at a section under the applied load. It is found that the loading may be increased from P_1 to 1.198 P_1, or about 20% without causing plastic stress anywhere except in the small shaded region over the middle support. With a group

of four equal loads applied at the one-third points of the spans, the corresponding increase is nearly 50 per cent.

It would seem that any modern or future stress theories which aim at economical and efficient use of materials should recognize the effect of shape of section on the ultimate strength in flexure, and that some parts of a structure may be stressed to the yield point, and even beyond, without impairing its usefulness. Such recognition would tend toward the accomplishment of Mr. Karpov's purpose which is "* * * the designing of structures in which the evaluated and actual safety factors are identical."

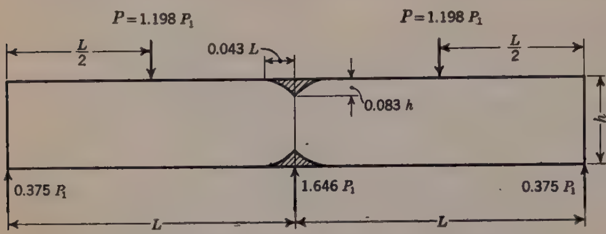

FIG. 50.—CONTINUOUS RECTANGULAR BEAM.

R. W. VOSE,[108] JUN AM. SOC. C. E. (by letter).[108a]—The statement by Mr. Brahtz that "the photo-elastic method is a two-dimensional analysis", must be taken to apply not only to the general nature of the problem, but also to every individual point of interest in the structure to be investigated by the method. It has not been generally appreciated that in many cases which are nominally two-dimensional—such as the bearing of rollers, the distribution of stress in riveted gusset-plates, and the concentrations of stress in the vicinity of sharp corners—third-dimensional stresses arise in the flat model, due to surface distortions and to methods of load application in the actual model. These stresses inevitably balk the investigator, since they affect his tool—the light ray—in an indeterminate manner, and force him to avoid many localized regions of high stress which may be critical in certain types of design involving repeated stress.

The same limitation is encountered when an attempt is made to analyze the surface stresses in a three-dimensional structure by means of any of the "surface photo-elasticity" methods, of which Mr. Brahtz mentions one. A sheet of any of the photo-elastically active materials may be conveniently examined from one side only, if the other side is provided with a reflecting layer to return the polarized light to the analyzing apparatus. If the reflecting side of this sheet is attached to, or inserted in, a loaded model of any material, the sheet will partake of the strains in the model, and the stresses induced in it may then be determined photo-elastically and used as an indication of the stresses in the model. The method fails in the general case, however, due to the three-dimensional nature of the stresses induced in the surface sheet by the model, and it is found to give accurate results only in regions where the stress is not changing rapidly, in which case strain-gages might equally well be applied. Further limitations in this particular application come from the low stresses induced in the photo-elastic model due to their

[108] Instructor, Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

[108a] Received by the Secretary October 26, 1936.

unusually low moduli of elasticity as compared with engineering materials, and from the present inability of the method to give other than the shearing stress at a point.

In addition to the usual transmission type of polariscope as described by Mr. Brahtz, the writer has found considerable use for a portable reflecting polariscope which has been designed at the Massachusetts Institute of Technology. In this instrument are combined an observing telescope of conventional size, mercury arc and incandescent illuminators projecting through the objective lens of the telescope, and a polarizing unit mounted over the objective lens and serving both as a polarizer and an analyzer. This is used with a specimen provided with a reflecting surface or a separate reflector, as mentioned previously, and owing to its small size may be carried to a model loaded in any suitable testing machine and moved from point to point around a model of any complexity. With its small size goes a correspondingly small field and low illumination, so that the instrument is not suited for projection work and must be used visually over small areas.

The same portable polariscope can be converted from the usual type of differential interferometer to an interferometer of the Fabry type with which the individual principal stresses can be determined. The principle of this interferometer and the equations for the solution of its indications are the same as those of the Mach-Zehnder interferometer which Mr. Brahtz describes, but instead of the expensive reflecting apparatus described by him the two surfaces of the model itself, polished to high reflecting power and parallelism, are substituted. This method promises to be an inexpensive supplement to other photo-elastic methods.

In connection with the determination of the individual principal stresses from the shear stress pattern furnished by the usual differential polariscope, it should be mentioned that each method has its own field of applicability. The point-to-point integration methods of Filon and of Neuber are dependent on having a number of free boundary points, or other points at which the complete stress conditions are accurately known; but on the other hand they are the only methods that can be applied to models which have flowed plastically. Such models are of importance in the fields of mechanical processing and soil mechanics where the possibilities of "photo-plasticity" have only been touched. On the other hand, the lateral extensometer methods, the Mach-Zehnder and Fabry interferometric methods, and the membrane methods all depend upon elasticity in the model.

In reviewing the past applications of photo-elasticity it seems that the field of mechanical engineering has been rather thoroughly covered by the present methods, and the designer in this field now has available sufficient data on stress concentrations and similar effects to enable him to handle, satisfactorily, most questions dealing purely with stress The overshadowing influence of fatigue, surface conditions, vibration, residual stresses, and many other factors, lessens the importance of accurate stress estimation. In contrast, the structural engineer is faced with problems in which stress calculations are of major importance in the design, and here it is that photo-elasticity may find increasing fields of usefulness in its present form.

RAYMOND H. HOBROCK,[109] ESQ. (by letter).[109a]—Considerable information on the mechanical and physical properties of the aluminum and magnesium alloys is presented in the paper by Messrs. Jeffries, Nagel, and Wood. The authors have wisely indicated quite a number of conditions for which it is best to consult the manufacturer of the metals before the selection of a particular alloy is made. It is the latter point which the writer intends to emphasize.

Civil engineers, in common with a majority of other engineers, are probably most familiar with the ferrous alloys and, through training and association, have become familiar with the interpretation of data on mechanical properties with the ferrous alloys in mind. The familiar mechanical properties for which values are usually given are the modulus of elasticity, the yield stress, the ultimate stress, the percentage elongation, percentage reduction of area, and, more recently, the endurance limit. If the engineer proceeds, by the same testing methods, to gain numerical values for these quantities for an aluminum alloy and for an iron-carbon alloy, the question arises, first, as to the legitimacy of a comparison of such numerical quantities as a basis for the determination of the general useful characteristics of the materials for specific applications. Secondly, one may inquire as to what other information is required for a reasonably complete analysis, providing a comparison of the measured numerical quantities of the mechanical properties are admitted as a legitimate first approximation for an estimation of usefulness.

In order to gain some idea of the co-relation between the usual measured mechanical properties and the information which might be required by a civil engineer, it is important first to understand what the quantities determined for the mechanical properties actually measure, and then to examine whether or not the properties so measured are pertinent in the design of a successful structure.

Of the mechanical properties usually measured:

(1) The elongation, the reduction of area, and the contraction measure the capacity to flow; that is, they measure the plastic characteristics;

(2) The elastic limit and the yield stress give values at which the plastic characteristics begin; that is, they give numerical values for forces required for the material to deform or to flow some small amount fixed by general agreement;

(3) The ultimate strength is a specific stress value measured during the course of the plastic deformation; it measures the force required for plastic flow at the point where the flow presents the greatest resistance; and,

(4) The endurance limit is a value for a specific stress which may be applied any number of times in complete reversal without causing failure.

Perhaps the most striking fact about an examination of these mechanical properties is that, with the exception of the endurance limit, they are all

[109] Director, Research Laboratory, Bundy Tubing Co., Detroit, Mich.

[109a] Received by the Secretary October 26, 1936.

associated with the plastic characteristics of the material. Since few structures are designed with the intention of causing the structure to undergo a plastic deformation, some doubt may arise as to the value of such quantities in the design of rigid structures. The value does lie chiefly in that these measured properties define a region that must surely be avoided. The yield stress or the elastic limit stands at the indefinite portal to this region—they reveal that at this point the material "flows a little"—how little is a matter that must be further investigated.

Nevertheless, the engineer has learned to make use of these quantities, and he learned them, perhaps, very largely in association with the ferrous alloys. He may express his doubt of the exact location of the beginning of the region in which the material "flows a little" by being on the safe side and assuming that that stress value is 20% too high; and he usually makes his structure two to ten times as strong as his best calculations (based upon the expected load and the yield stress of the material) indicate that it need be—he uses a safety factor. That safety factor can, and is, properly intended to compensate for more than accidental or uncontemplated excess loadings, or the imperfect state of current knowledge of stress analysis. It attempts to take into account a great many of the characteristics of the material that are unknown to the engineer or that are not quantitatively defined and subject to mathematical treatment.

With this in mind the first question may be expanded and restated as follows: If it is conceded that behavior characteristics are usually taken into account in structures of familiar ferrous alloys by the introduction of safety factors learned through much experience, are all these characteristics so inter-related in the case of the light structural alloys that the same safety factors might be used as in the ferrous alloys? If not, are there safety factors that will serve such purpose? Or do the light alloys have some characteristics quite different in relative importance from the same characteristics for a ferrous alloy that should receive special consideration in the design of structures?

It may be of value to examine a few of the conditions that are normally imposed upon a structure, or that might reasonably be expected to be imposed upon a structure during the course of its life, and then inquire into what behavioristic characteristics are indicated for determining the relative value of materials in respect to these requirements.

Anticipated Loadings.—Anticipated loadings of both a static and dynamic nature may be approached in the manner described by Mr. Karpov, making use of the best methods of computing stresses and based primarily upon the modulus of elasticity, the yield stress, and the fatigue properties.

Accidental Localized Overloads.—Such an accidental overload may result, for example, from the impact of an automobile or other vehicle upon a structural element of a bridge. Under such conditions it is valuable to know how the material of the structure, and, finally, how the structure itself, will re-act toward the redistribution of stresses. Will the structural element fail readily under an impact load? How sensitive is the structural material

to boundary conditions, to stress concentrations, such as rivet holes, surface injuries, corrosion cracks, etc? Will the sudden application of a load cause failure at loads much less than if slowly applied? The answers are dependent upon the capacity of the material to undergo a plastic deformation quickly and, therefore, to redistribute the stresses. Some idea of the relative values of different materials in this respect is to be gained from impact testing, especially at various impact velocities. A more fundamental material property which perhaps serves as a measure of dynamic ductility is the damping capacity, that is, a measure of the capacity of the material to absorb energy.

Much work still needs to be done in the correlation and interpretation of values for damping capacities directly in terms useful to the design engineer. Indeed, the first need is to devise experimental means for measuring the damping capacity as a function of applied stress. At present, values for damping capacity are usually correlated with the maximum stress in the specimen only, except for very small stresses, as in the case of a free bar subjected to small longitudinal vibrations.

Relaxation or Creep.—If a sheet of metal contains a small hole at a distance of at least three times the diameter from the edge of the sheet, and if this sheet is subjected to tension forces, a stress concentration occurs on the circumference of the hole and on an axis of the sheet perpendicular to the direction of tension which is three times as large as the stress in the undisturbed part of the sheet. However, if the hole is filled tightly with a rivet, in such a manner that radial forces are exerted on the circumference of the hole, these stress concentrations may be eliminated. Suppose that a rivet is properly driven or squeezed in a hole and then exerts the radial force necessary to avoid the stress concentration, the question arises as to how long this condition can be maintained; that is, will the material relax after a certain length of time and relieve the radial forces? If it does, it will result in re-establishment of the stress concentrations. Obviously, information on relaxation and of creep, as well as the results of practical experiments, would be of value in this important case.

Properties at Elevated Temperatures.—These properties are of importance to bridges, buildings, and structures of this type in so far as they aid in predicting the behavior of the structures under unusual conditions as, for example, in the case of a local conflagration in a building. If the strength properties of a structural material decrease rapidly with increase in temperature, it may happen that a purely local fire will endanger the entire structure. It would seem that, because of their relatively low melting points, the aluminum alloys should show a much more rapid decrease in such strength properties with increasing temperature than a ferrous alloy. Furthermore, those aluminum alloys dependent upon precipitation-hardening effects for a goodly portion of their strength, might be expected to decline in strength rapidly at relatively low temperatures. On the other hand, the aluminum alloys possess a great advantage in their high heat conductivity, this being about 4.5 times as great as for low carbon steel at 100° C. It

would be of considerable interest, however, to have comparative data on structures of the two materials designed for the same loading and subjected to identical conditions of local heating.

The Course of Destruction of a Structure.—One type of destruction, obviously, is started as a structure is built; that is, the natural deterioration. For a proper consideration of this factor, it is necessary to consider the corrosion properties in relation to the corrosive agents likely to act upon the material of the structure, the fatigue properties, the creep, the influence of the expected atmospheric temperature variations, and, in the case of precipitation-hardened materials, the influence of time itself. Since the acquisition of the highest strength properties in the case of some of the precipitation-hardened materials is a function of time after the solution heat treatment, one might expect that deterioration in properties might also be a function of time. This can be shown to be true in tests at elevated temperatures. From theoretical considerations one might expect a precipitation-hardened material to be thermodynamically more unstable at room temperatures than at elevated temperatures so that the tendency for a decrease in strength properties is greater at room temperatures. However, it is quite certain that the rate of approach to the condition of a thermodynamically lower energy level (and, consequently, to decreased strength properties) is much less at the room temperature. Information regarding strength properties of the light alloys after long times, under stress, and subject to the usual temperature variations of the atmosphere, would do much toward deciding whether or not this is a consideration of any practical importance.

In the event that a structure does fail by overloading, or a part of it fails in fatigue, and that this local failure leads to failure of the entire structure, how will it fail? Will it fail suddenly or will there be some warning, such as an apparent slow deformation of the structure? Obviously, some clue of this is to be gained by comparison of the plastic characteristics of the material—exactly those properties most often measured and listed.

Let no one gain the impression that all this desirable information is available about structural steel and is not available about the light alloys. This is certainly not the case; but engineers have used structural steel for a long time, and there is incorporated in the safety factors which they apply, some estimation of many of these unknown characteristics. The future will undoubtedly see a considerable use of the light alloys for civil engineering, and it has been the writer's intention to indicate that values for the ordinarily measured behavioristic characteristics, coupled with the usual safety factors—or perhaps with any one or two safety factors—will very likely not be sufficient to produce a good design.

At the present state of knowledge and in consideration of the relative unfamiliarity of structural engineers with the light alloys, the advice of Messrs. Jeffries, Nagel, and Wood to "consult the manufacturers" should be strongly emphasized.

WILLIAM F. CLAPP,[110] ESQ. (by letter).[110a]—The money spent in the effort to solve the various angles of the corrosion problem, with the ultimate object of producing superior metals, is staggering. Enormous numbers of long-time exposure tests are being conducted in many parts of the world under all conceivable conditions. On the other hand it seems that intensive studies of service records either have not been equally stressed, or the results have not been published or made available to the interested student. Therefore, it is to suggest that additional effort be made to analyze carefully the various factors that have resulted in the innumerable examples of exceptionally good and exceedingly poor service records of the various metals. It is not within reason to anticipate that findings of value, comparable to those obtained from exposure tests, might result from the equally intensive laboratory studies of service records.

Although deeply interested in the various factors connected with the problems of corrosion, the writer's observations have been entirely confined to the study of a limited number of examples of metal deterioration, in which one or more of a large group of marine organisms has been designated as either directly responsible for, or as producing indirectly the conditions favorable for, accelerated corrosion. To read in recent engineering reports that the "marine borers" were responsible for excessive deterioration in certain steel structures in California; and in other reports that probably certain species of *Balanus* accomplished the same results by means of some powerful secretion, is certainly sufficient to arouse one's curiosity. Although the destructive powers of the marine borers should not be under-estimated, it is scarcely fair, when causes are difficult to find, nonchalantly to blame the "bugs." Some effort should be made either to prove or to remove the implication that marine organisms, particularly marine borers, are contributing factors of major importance. To prove them entirely innocent of this particular outrage is not as simple a matter as at first anticipated. Dr. F. N. Speller and others have indicated[111] some very insidious methods by means of which the marine organisms may after all contribute their share to metal corrosion. However, actual proof seems still to be lacking.

An accumulation of records of the biological, chemical, and electrical conditions surrounding outstanding examples of excellent and poor service, together with the analyses of the metal involved, would not be an appalling program of research, particularly when the annual losses due to corrosion, as described and tabulated by Mr. Aston and others, are taken into consideration.

J. C. HUNSAKER,[112] ESQ. (by letter).[112a]—Three stimulating papers on the application of stainless steels, aluminum alloys, and magnesium alloys have been presented by Messrs. Ragsdale, Hartmann, and Winston, respectively. The three papers will be considered together by an engineer faced with the

[110] Cons. Biologist, Duxbury, Mass.

[110a] Received by the Secretary October 26, 1936.

[111] "Corrosion—Causes and Prevention", by F. N. Speller, *Transactions*, A. S. M. E., June, 1935.

[112] Prof. of Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

[112a] Received by the Secretary October 26, 1936.

necessity for making a decision as to the materials for his design. He will appreciate Mr. Hartmann's paper best, perhaps, because it gives him practical design data rather than physical properties and generalities which, although significant, may not bear on his specific problems. Obviously, there are other criteria than strength, weight, and cost of the metal involved in structural or mechanical design. The engineer responsible for a decision must be assured of durability and maintenance, procurability in quantity without sacrifice of uniformity, shop and erection tooling and practice, and other factors which bear on over-all safety. There is less information offered concerning the latter factors, as applied to stainless steels and magnesium alloys.

The engineer, for example, may be interested in the possibility of light-weight floors for existing bridges, if the substitution of such a floor will prolong the life of the bridge; but he is loathe to abandon conventional materials. It is possible that such floors could be built of stainless steel with satisfactory weight, life, and durability, but the cost of material and fabrication would be high. Since neither rolled nor extruded sections are available, all beams, stringers, and secondary members must be built up. They could not be welded to the existing structure, in all probability, since torch-welding affects the stainless steel and shot-welding is out of the question on thick members of mild steel. It would be difficult to drill them in the field so that they could be riveted, and riveted-joint design of thin members is uncertain and complicated. Repairs to existing stainless steel structures would appear to be difficult. Moreover, for bridge loads, substantial sheets would be required instead of the light gages used in rail cars and deck houses. These sheets are well-nigh impossible to shear and are difficult to form as they have considerable "spring-back." If bent too sharply there seems to be an effect on the grain structure due to the excessive cold working, with the result that the corners corrode. From these considerations one might conclude that stainless steel, as utilized at present, would not be economical, although its durability and low maintenance costs tend to offset the higher costs of the material itself.

A real virtue of stainless steel lies in the indirect savings it makes possible for low-weight, high-speed means of transportation. The light-weight train operating at 90 miles per hr can do so on lighter rails and bridges than is possible with standard equipment. Duralumin or magnesium can be considered as equal to stainless steel from this point of view, although one may question both from the standpoint of fire hazard, particularly the magnesium alloys. The latter are also questionable from the standpoint of corrosion, especially in climates burdened with salt moisture or chemicals.

From what the writer has observed of the corrosion of duralumin in contact with steel in airplanes, he would question Mr. Hartmann's recommendation that steel rivets and pins be used for connections on structural aluminum. With adequate paint, corrosion can doubtless be prevented, but all one has to do is to look around at bridges these days to see how little paint they get. Such connections are questionable, particularly near the coast where the fogs are salt, or over railways where the blast from a locomotive blows sulfurous gases against the structure.

In buildings, the criterion for stiffness may be the plastered ceiling which has a definite deflection limit if cracks are to be avoided. It is true that aluminum beams can be made deeper so that they will have the required stiffness, and yet be lighter than steel; but when fireproofing is added, the total weight is as great as, or greater than, that of the steel. Little or nothing may be saved in the weight of the building. Magnesium or stainless beams probably suffer from the same lack of stiffness; the one due to a low value of E, the other due to the use of deep, thin-walled elements carrying high unit stresses.

The engineer needs to know the limits of available standard elements of construction for stainless steel and magnesium. Stainless sheets seem to be limited in width because of the difficulty of rolling a hard metal to a constant thickness. The writer does not know what the limits are for the various gages; he assumes that magnesium is not available in long extruded sections having larger areas. Aluminum alloy was in this condition before 1926. Stainless steel can be formed through rolls so that long sections are available, but they are limited in width. It is difficult to know what can be done with tubes but the structural engineer considers this section an impossible one in any case, since he cannot fasten other members to it conveniently.

In general, the three materials—stainless steel, aluminum, and magnesium—are clearly of great interest both to the structural, and to the mechanical, engineer; but the specific applications will be approached with caution and step by step. It is fortunate that development of the three types of metal is in such competent hands, but it will require both experience and opportunity before engineers are able to proceed with confidence to the special application of each to uses in which it has a real advantage.

Horace C. Knerr,[113] Esq. (by letter).[113a]—A symposium on light-weight structural design seems incomplete without including that class of material which affords the lightest possible construction, namely, heat-treated alloy steel, in the range of 150 to 200 kips per sq in.

Considered in terms of load-carrying capacity for a given weight of material, metals may be compared on the basis of a strength-weight factor obtained by dividing the design strength value (that is, ultimate strength or yield strength), in kips per square inch, by specific gravity.[114]

Taking representative strength values for magnesium and aluminum alloys, respectively, and comparing them with steel on this basis:

$$\text{Magnesium} = \text{Duralumin} = \text{Alloy Steel} = \text{Strength-Weight Factor}$$

$$\frac{35}{1.8} = \frac{55}{2.8} = \frac{150}{7.8} = 19$$

If the foregoing strength values are taken as ultimate, they are conservative for the metals indicated. If they are regarded as yield values, the factors for magnesium and duralumin are extreme, but that for steel can be

[113] Cons. Metallurgical Engr.; Pres., Metlab Co., Philadelphia, Pa.

[113a] Received by the Secretary November 4, 1936.

[114] "Material Selection on Strength-Weight Factors", by H. C. Knerr, *Automotive Industries*, April, 19, 1923.

exceeded. Steel, therefore, as a material, may equal or exceed in lightness the so-called light alloys.

Steel lends itself admirably to assembly by any of the welding processes—acetylene, electric arc, electric resistance, etc.—whereas the magnesium and aluminum alloys do not.

The autogenous welded joint is more efficient as regards material than the riveted joint and, therefore, it is lighter, thus contributing further to the lightness advantage of steel.

Comparison of the high-grade, heat-treatable alloy steels with the stainless steels, such as Alloy 18:8 (which derive their strength from cold work), shows certain advantages for the less costly material.

Stainless steels can be welded only by a proprietary process without sacrificing strength and corrosion resistance. They can be given high strength (150 kips per sq in., or more) only in the form of thin ribbons or strips and must be formed and fabricated in this hard, springy condition—a difficult feat.

Their necessarily thin structural sections and the relatively low elastic modulus of the material tend to result in flimsiness and excessive deflection under load.

Their costliness is somewhat offset by their corrosion resistance and attractive appearance, and, in some cases, by their non-magnetic character.

Heat-treatable alloy steels, such as chrome-nickel, chrome-vanadium, chrome-molybdenum, and others of moderate cost, can be used as forgings, rolled or pressed shapes, or, desirably, as seamless or welded tubing which possesses outstanding engineering advantages. These steels are easily worked, hot or cold, and some are better adapted to welding than others.

Parts may be heat-treated prior to assembly by means of bolts or rivets; or, units of moderate size, after assembly, by welding, provided necessary precautions are taken to avoid distortion. The weld is strengthened by the heat treatment.

Magnesium, aluminum, and stainless steels have all been brought to their highest point of development in the construction of aircraft. The same is true of heat-treatable alloy steels. These steels, in tubular form, assembled by acetylene welding and then heat-treated, have been used successfully for many years in the most vital parts of aircraft, such as the landing gear, wing beams, and heavily stressed structural members where great strength, toughness, reliability, and resistance to shock and fatigue, combined with lightness, are essential. They are still unequalled by the more recently developed materials for these locations.

The three types of metal discussed in this Symposium are all more or less subject to proprietary interest. Their engineering and industrial applications have been promoted vigorously by their sponsors, chiefly in competition with the ordinary steels of low strength.

The heat-treatable alloy steels of high strength, being common property, have lacked this sponsorship and, consequently, have been rather neglected in the rush of new developments. They offer advantages of great importance in the field of light-weight construction, advantages to which the Engineering Profession may well give careful attention.

F. T. SISCO,[115] ESQ. (by letter).[115a]—Such a clear and comprehensive picture of the alloy steels and the light non-ferrous alloys has been presented in this Symposium that it seems impossible to add anything of interest on the properties and applications of these materials. There is one point, however, which was neglected by all the authors, which should be mentioned. That is, the rôle played by the research laboratories in developing these alloys so that the engineer will have readily available, at a reasonable cost, a material with certain distinctive properties or combination of properties. Although the writer will confine his remarks to alloy steel they are, perhaps, equally applicable to the non-ferrous alloys.

The history of alloy steels goes back scarcely fifty years, but in that short time splendid results have been attained and some remarkable materials have been produced. Considering the vastness of the field, it is quite astounding that the research laboratories have produced steels which meet almost every present industrial demand. The vastness of the alloy-steel field may be realized when it is remembered that of the alloys of iron with carbon and the eight common alloying elements about 50 000 000 combinations are possible.

Broadly, there are two kinds of research on alloy steels. It is difficult to give them names. They might be called scientific and practical; or if one wishes to be as cynical as a certain physicist, one might call them planned and "hit-or-miss", or "machine-gun" research. Under the research which the writer has tentatively termed "scientific" or "planned" is included work on constitutional diagrams and its ramifications. There are many outstanding examples of such work by American metallurgists. This type of research has as its object the addition of basic knowledge on alloy systems and may or may not have practical consequences. No metallurgist will deny, however, that it is important that the gaps in basic knowledge be filled as soon as possible, even if nothing practical results.

Practical or "hit-or-miss" research is essentially carried on by using as a base material a commercial carbon steel or cast iron, and adding special alloys, or treating the material in special ways, to note what happens. For many years most of the research on alloy steels was conducted in this manner, and most of the common alloy steels are the result of such work. There is much justification for it, because the effect of combinations of alloying elements can rarely be predicted from the known effects of these alloys when used singly. Much of the research at the present time is a combination of planned and "hit-or-miss" methods.

One of the important essentials in beginning research work on alloy steels, whether it is to fill a gap in basic knowledge or to develop a new low-alloy steel, is for the metallurgist to be sure that he is not repeating work already done by some one else. Despite the fact that only a few hundred, or at most a few thousand, of the 50 000 000 possible combinations of alloys have been investigated, the literature on alloy steels is enormous. Reports of such research are scattered through thousands of journals and books in many

[115] Metallurgist and Editor, Alloys of Iron Research, The Eng. Foundation, New York, N. Y.

[115a] Received by the Secretary November 17, 1936.

languages. Reviewing this literature is time-consuming and, except where large libraries are available, practically impossible. Consequently it has frequently happened in the past that, after a metallurgist had completed his research, he found that similar work had been done previously by some one else.

To avoid such unnecessary duplication of effort "Alloys of Iron Research" was founded in 1929 by The Engineering Foundation. Its object is to review this vast literature, "weed out" the clearly erratic work, correlate and summarize the remainder, call attention to conflicts in data, and recommend research to fill gaps in basic knowledge. The results of this critical review of the world's research are correlated in a series of monographs which give to the metallurgist all the essential known data on alloy steels, thus enabling him to plan his research and development work with no loss of time and effort. They also give to the engineer an unbiased summary of the properties of all the alloy steels known to-day.

What will be the future of alloy steels is any one's guess. It has been said that the world is entering an alloy-steel age. No one knows now how many of the 50 000 000 possible alloy steels will be worthless commercially or at least no better than the ones available to-day. No one knows how many of them will have remarkable properties, so remarkable perhaps that a veritable revolution in construction may occur when they are developed. The most that can be said is that good alloy steels are available to-day (steels which are entirely satisfactory for many commercial applications) but that more research is needed to develop new steels, to improve the properties and reduce the cost of the older ones. That the iron and steel industry recognizes this is plain from the speed with which its laboratories are working, from the rapidly expanding literature reporting the results of research, from its splendid co-operation in the basic research to fill the gaps in the knowledge of alloy systems, and from the new alloy steels which are made available each year.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE MODERN EXPRESS HIGHWAY

Discussion

By Messrs. Elmer R. Haile, Jr., H. W. Giffin, and T. T. Wiley

Elmer R. Haile,[12] Jr., Jun. Am. Soc. C. E. (by letter).[12a]—Since more and more monies are being expended for the renovation of obsolete highways, this paper comes at an opportune moment. It succeeds admirably in outlining the subject of building safety into highway design, and in suggesting desirable researches and tests. This discussion presents the results of experiments pertaining to various design details, refutes the theory that the design of highways for high speeds will lower the accident rate, and suggests an alternate method for lowering the accident rate.

The Design Speed.—The design speed of 100 miles per hr appears to be unnecessarily high. It may be true that automobiles can be built to cruise at 100 or even 120 miles per hr, but that fact in itself does not justify the design of highways for those speeds, because the average driver will never travel that fast. Conceivably, a few drivers might develop high speeds, but their numbers would be far too small to warrant the additional expenditure involved.

Studies indicate that the intelligence of the human race has not increased measurably in the last 100 or 1 000 yr. Hence, it is erroneous to assume that the ability of Man to reach quick decisions or that his tendency to make mistakes will change appreciably in the near future. To quote from the paper, "the present railway maximum speed is approximately 80 miles per hr. The drivers of trains are trained, picked men, familiar with their runs, * * * with definite orders and under close supervision." Even so, there are instances of trainmen disobeying orders and signals. If men with only average judgment, slight supervision, little training, and perhaps no knowledge of road conditions were allowed to take over the operation of trains, the number of accidents would "sky-rocket." On the highways the personnel is exactly as described in the preceding sentence, with the additional formidable handicap

Note.—The paper by Charles M. Noble, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Fred Lavis, Joseph Barnett, G. E. Hawthorn, John F. Fairchild, Leslie R. Schureman, and C. H. Purcell.

[12] Junior Highway Engr., U. S. Bureau of Public Roads, Gettysburg, Pa.

[12a] Received by the Secretary October 26, 1936.

that their vehicles do not run on fixed rails. The path of each car is controlled by the driver and, therefore, may be subject to every possible human reaction. Whereupon it may be reasonably deduced that highway speeds ranging close to the railway maximum of 80 miles per hr are out of the question for the average driver.

A speed of, say, 60 miles per hr might be safe, but even in this case it would be necessary to bar incompetent drivers from the road. In practice, this proves to be almost impossible. There are always those drivers who pass the most rigid of examinations and then proceed to be reckless, or at least lax, in concentrating their attention on driving; not to mention those who become incompetent due to ennui and fatigue, and intoxication.

All of which leads to one conviction, which is, distasteful though it may be to many persons, that the solution is to fix a maximum speed limit. Granted that there are some drivers who know what a safe speed is, and can safely "hit 90", yet for every such driver there are a dozen who can not judge whether or not they are proceeding at a safe speed until an emergency arises. Likely as not it is an innocent party who pays with his life for the driver's poor judgment. The proposed speed limit must be enforced, in sharp contrast with most present-day limits. Of the several means of enforcement, schooling is undoubtedly the best in the long run, supplemented with police patroling and, in some cases, mechanical speed control.

After exhaustive tests and observations are made and the maximum safe speed is determined and agreed upon generally, then the minimum design standards for that limit may be established. Lower speed limits will be necessary for secondary roads, and for trunk roads in congested areas and mountainous country. Of course, there is no harm in building a highway safe for speeds greater than the speed limit; such construction increases the factor of safety. The point which the writer wishes to emphasize is that the construction of 100 miles-per-hr highways will not in itself serve to reduce the number of fatal accidents if cars are permitted to attain 100 miles-per-hr speeds. There is a certain speed above which the average driver can not go without appreciable risk of a fatal accident, no matter if the road is theoretically safe for 120 miles per hr. For example, statistics from several sources indicate that the majority of present-day accidents occur in the country, during the day, on dry pavements, and where the road is straight. These accidents are inexplicable; they must be charged to the frailties of Man.

The standard of 100 miles per hr has another vitally important disadvantage, namely, the high cost, not only for the right of way and area of pavement, but particularly for the elimination of grade crossings and points of access, and, except in flat terrain, for the grading. Only a few of the heavily traveled arterial routes can qualify as being sufficiently important to justify such expenditure.

The writer is not prepared to suggest a maximum value for the design speed, but he does conclude that it need not be very much higher than one-half the design speed suggested by Mr. Noble.

Accidents.—Referring to Table 1 in the paper, it is noted that more than one-third of all accidents involve pedestrians. These accidents cannot be

reduced by improvement in road design, except by the construction of sidewalks, which, incidentally, must have a surface equally as good as, or better than, the roadway along which they are built or they will not be used by those they are intended to serve. About one-half the pedestrian accidents are due to the carelessness of the pedestrian. Education, particularly in the schools, is reducing that cause of accidents. The other half of the accidents that are blamed on the driver cannot be reduced by building high-speed roads.

Again, referring to Table 1, it is noted that less than 7% of the accidents involve collision with fixed objects. About one-half of these accidents occur on curves, and one-half, on straight sections. Therefore, an improvement in radii of curves will not materially reduce the accidents due to vehicles leaving the roadway, and it may increase them by encouraging higher speeds on curves.

Again, it is noted that about 45% of the accidents involve collision with other automobiles. Of these collisions, about 11% occur at intersections, 9% are "head-on" collisions, and the remaining 25% occur with vehicles going in the same direction. The 11% part may be reduced by the proper development of intersections. The majority of all intersections, even in congested areas, will probably remain at grade for economic reasons. Much study is necessary to determine some type of intersection that will not be inherently hazardous. For example, islands or circles inserted in high-speed roads may be very dangerous. Higher speeds than those prevailing at present will certainly increase the number of accidents at grade intersections, regardless of their treatment, because of the unavoidable crossing of traffic. The 9% part—"head-on" collisions—may be reduced by the use of 20 or 22-ft roads instead of the usual 16 or 18 ft, by providing ample sight distances, by widening and easing of curves, and, in the few cases where four-lane construction is economically justified, by the use of divided roadways. However, since the great majority of trunk highways (about 95%) will remain two-lane, two-way roads, speeds higher than the present average will increase "head-on" collisions more than the improved design will reduce them. The 25% portion of all accidents can be reduced in about the same manner as the 9% just mentioned. In one instance, statistics show that accidents involving vehicles proceeding in the same direction are fewer on divided highways than on two-lane, two-way highways. yet, in another instance, the accident rate on the new divided highway is greater than the rate on the old two-lane, two-way highway, due possibly to the higher speeds obtained on the improved road.

To summarize, an analysis of the various types of accidents indicates that improvements in highway design will not serve to lower the accident rate as long as correspondingly higher speeds are permitted to develop, thus demonstrating the futility of trying to design roads for the maximum speeds of which modern cars are capable.

The Dual Highway.—The dual highway is undoubtedly indicated wherever a two-lane highway is inadequate in capacity. In at least one instance, the concrete slabs of one-half of a new four-lane highway have been jacked up and moved transversely so as to leave a neutral area between cars traveling in opposite directions.

The writer has observed a serious disadvantage in having a width of pavement in excess of approximately 22 ft, especially for a two-lane, two-way highway. Many drivers use the road as a three-lane highway, despite the narrow clearances, making it even more dangerous than the 30-ft, three-lane design.

Superelevation.—Past practice has been to superelevate according to rule of thumb, or not at all. Consequently, the driver never knows what to expect when he encounters a curve. The greatest danger lies in having a series of curves superelevated for a certain speed followed by a curve superelevated for only one-half that speed, as would be the case if the first-mentioned curves were banked to the limit, and the following curve was very much sharper, but not banked any higher, inasmuch as the limit had already been reached.

The writer has conducted road tests in an effort to determine the proper rate of superelevation for various combinations of speeds and radii. Modifying Equation (1) so as to take into account the transverse frictional force that is developed whenever the speed of the vehicle differs from the speed at which the gravity force due to the superelevation exactly balances the centrifugal force:

$$F + E = 0.067 \frac{V^2}{r} \qquad (16)$$

in which, in addition to the notation of the paper, F is the coefficient of friction between tires and road surface.

Observing the speeds at which drivers begin to encounter difficulty in keeping their cars under control on curves of known radii and rates of superelevation, substituting these values in Equation (16), and solving for F, values are obtained ranging from about 0.05, at which practically no centrifugal force is felt, to 0.50, at which a severe centrifugal force is felt, throwing passengers against the sides of the car, and resulting in considerable transverse creeping or skidding toward the outside edge of the curve. Upon plotting the coefficient, F, for numerous values of V, a wide band of points results. A curve drawn slightly below the average of these points is reproduced in Fig. 6, and represents the judgment of the more conservative drivers as to the allowable values of F for safe driving.

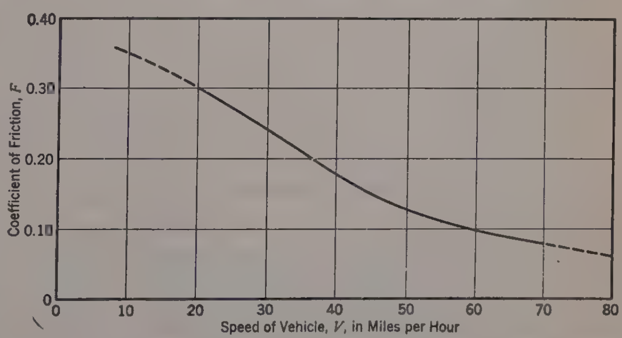

FIG. 6.—ALLOWABLE VALUES OF F FOR SAFE DRIVING ON CURVES.

An interesting feature of the graph is the fact that drivers do not hesitate to develop high values of F when traveling at slow speeds around sharp curves. However, a value of 0.30 for F still allows some factor of safety, for skidding does not occur under normal conditions until F reaches about 0.50. This margin

need not be large at low speeds when the car can be slowed down within a few feet. At the other end of the graph, it is noted that much lower values of F are obtained, resulting in larger factors of safety. This is desirable at high speeds when the results of skidding may be disastrous, and when a considerable distance is required for reducing speed in case the car should begin to skid, as might happen if the pavement were uneven, or slippery with mud or grease.

Table 2 gives the maximum allowable curvature for various maximum speeds the values of F being taken from Fig. 6, and the maximum value of E being assumed arbitrarily at 0.10 ft per ft of width. In sections where ice conditions prevail over a considerable part of the year, a lower value, say, 0.07, may be advisable, in which case the maximum allowable degree of curve would be somewhat less.

TABLE 2.—MINIMUM RADII FOR VARIOUS DESIGN SPEEDS, USING EQUATION (16) AND FIG. 6, AND ASSUMING $E = 0.10$

Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)	Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)	Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)	Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)
20	0.30	67	35	0.21	265	50	0.13	728	65	0.09	1,490
25	0.27	113	40	0.18	383	55	0.11	965	70	0.08	1,824
30	0.24	177	45	0.15	543	60	0.10	1 206	..	....	.....

Since, in general, the average speed of vehicles will not be as high as the maximum safe speed, all curves flatter than the limiting curve should be superelevated for the average speed, using Equation (1), taking care, however, that the use of a low value of E resulting from that computation will not cause a transverse friction factor, F, greater than the allowable for the maximum safe speed as computed with Equation (16).

Spirals.—The writer begs to differ with Mr. Noble's opinion that spiral curves are unnecessary. To illustrate, assuming curves of 2 900-ft radius with 1 000 ft of tangent between reversals, as suggested in the paper, and introducing spirals of 1 000-ft length (which will consume all the tangent), computations show that the offset required for inserting the spiral between the tangent and circular curve is 14.35 ft. This transition cannot be accomplished by a car within a 12-ft lane when spirals are omitted; the driver is forced to make his own spiral rather short in order to stay within the lane. Such an abrupt spiral requires special concentration and judgment, which is foreign to the ideal stated in the paper that vehicles should enter the curve smoothly, without tendency to be thrown in or out—almost automatically. When the spirals absorb all the tangent, as in the preceding illustration, there is no break in curvature—the driver is turning his steering wheel at a constant rate between the curves; but with a piece of tangent between the curves, the driver must adjust his car to run on the tangent for a few seconds, and then adjust his car to the next curve.

Contrary to general belief, the highway with the longest possible tangents is not the ideal to be sought. A flowing alignment is much more pleasing and restful to the traveler than monotonous "straightaways" followed by abrupt (although under the maximum degree) curves. This suggests the use of very long spirals between tangents and circular curves. The ideal treatment of a sharp curve, especially if it is sharper than the standard, is to introduce a spiral (or perhaps a series of other curves) of gradually increasing curvature for a sufficient distance ahead of the curve to enable the driver to release his throttle automatically anl reduce his speed enough to make braking unnecessary. In addition, sharp curves should be signed to indicate their safe travel speeds, for the benefit of that great number of drivers who are not gifted with sufficiently accurate judgment. The foregoing treatment is far safer than the usual method, which is to design a road for high speed until a certain under-pass or topographical feature is encountered and then promptly to forget the standards.

In going around a curve, whenever a vehicle develops a lateral coefficient of friction, designated by F, the body tilts with respect to a line normal to the axle. Upon entering the curve, it is advisable the develop the factor, F, gradually; otherwise, there will be discomfort due to sudden tilting. Since F varies directly with the degree of curve, a transition with uniformly increasing curvature would allow a uniform increase in F. A spiral curve has the property of uniformly increasing curvature; therefore, it is used for the purpose.

If C is the rate of change of acceleration toward the center of the curve, L_s is the length of spiral, in feet, V_s is the velocity of the vehicle, in feet per second, r is the radius of the circular curve, in feet, and, if the curve is not superelevated:

$$C = \frac{V^3_s}{L_s\, r} \quad \text{.........................(17)}$$

Tests conducted by the writer indicate that: (1) A value of C less than 3 ft per sec per sec per sec does not cause a noticeable lurch of the vehicle; (2) a value greater than 6 does cause a noticeable lurch; and (3) a value of C as great as 10 ft per sec per sec per sec causes a bad lurch. Substituting 3 for C in Equation (17), and solving for L_s:

$$L_s = 0.333\, \frac{V^3_s}{r} \quad \text{.......................(18)}$$

Changing to miles per hour, and substituting $\frac{F + E}{0.067}$ for $\frac{V^2}{r}$:

$$L_s = 15.7\ V\ (F + E) \text{....................(19)}$$

On an unbanked curve, $E = 0$ and Equation (19) reduces to

$$L_s = 15.7\ V\ F \text{.......................(20)}$$

On a banked curve, the part of the centrifugal force represented by E is balanced out by the gravity force due to the banking, and does not operate to tilt the vehicle with respect to a line normal to the axle; therefore, Equation (20) holds good for both banked and unbanked curves.

This formula gives the minimum length of spiral required for any combination of maximum safe speed, radius of curve, and superelevation. Upon applying the formula in practice, however, it develops that the length of spiral as computed is too short because of other considerations. If, for example, $V = 60$; $E = 0.10$; and $r = 1\,206$; whereupon, $F = 0.10$, it is found that $L = 94$ ft. As shown subsequently, the minimum length of transition from an unbanked section to a banked section, when $V = 60$ and $E = 0.10$, is 240 ft, with a recommended length of twice that amount. In all cases, the spiral should have the same length as the superelevation transition in order to provide uniformly increasing curvature simultaneously with uniformly increasing superelevation Therefore, Equation (20) may be discarded as superfluous.

Superelevation Transitions.—As a vehicle proceeds over a superelevation transition, it rotates about a longitudinal axis. This rotation, or angular velocity, or change per second of the rate of superelevation, may be expressed by $\frac{V_s}{L_s}(E_2 - E_1)$, in which V_s is the rectilinear velocity, in feet per second; $E_2 - E_1$ is the difference in rates of superelevation at the ends of the transition; and L_s is the length of the transition, in feet.

In the case where the superelevation increases uniformly from E_1 to E_2, the angular velocity jumps from zero to a certain value at the beginning of the transition, maintains this value throughout the length, L_s, and then drops back to zero at the end of the transition. This causes the vehicle to lurch at each end of the transition, because the change in angular velocity is very abrupt.

The change per second of angular velocity, or angular acceleration, may be expressed by $\frac{V_s}{L_v} \times \frac{V}{L_s}(E_2 - E_1)$, in which L_v is the length of the vertical curve at each end of the straight-line transition. The length of this vertical curve, or "rounding", may be as great as L_s, in which case there will be no tangent left, or it may be any length shorter than L_s. The writer has used $L_v = \frac{L_s}{2}$, although he cannot prove that that proportion is the best. Substituting $\frac{L_s}{2}$ for L_v, the expression for angular acceleration becomes, $\frac{2\,V^2_s\,(E_2 - E_1)}{L^2_s}$.

Experiments made by the writer show that an angular acceleration greater than 0.06 per sec per sec per sec produces an uncomfortable lurch, but that

one-half that value, or, say, 0.027, is satisfactory. Substituting that value for the angular acceleration and solving for L_s:

$$L_s = V_s \sqrt{\frac{2\,(E_2 - E_1)}{0.027}} \qquad (21)$$

which, changing to miles per hour, reduces to:

$$L_s = 12.62\, V \sqrt{E_2 - E_1} \qquad (22)$$

If, for example, the maximum safe speed is 60 miles per hr and the change in superelevation is 0.10, the required length of transition is found to be 240 ft, with 120-ft vertical curves. At other maximum safe speeds, with the same difference in superelevation, the required lengths are in direct proportion. At a reversal of two sharp curves, if $V = 60$ and $E_2 - E_1 = 0.20$, the length, L_s is found to be 340 ft. This is almost one-third less than the 480 ft, which would be used if it were assumed that the length of transition should be proportional to the change in superelevation. This is an advantage in locations where the tangent distance is limited, which is probable where the curves are sharp enough to require the maximum allowable superelevation. On the other hand, if the tangent is not limited, and if the introduction of a longer transition is feasible, the writer suggests that double the values of L_s given by Equation (22) be used. When the transition is quite long, the rounding at its ends may be an unnecessary refinement, in which case a straight-line transition may be used.

Grades.—Safety on icy pavements does not justify limiting grades to 5%, assuming there are no other considerations making such a limit desirable. The extra cost of grading or the disadvantages of the more tortuous alignment necessary to provide a 5% grade may far exceed the accumulated cost of spreading sand or cinders over the icy pavement from time to time during the winter months.

If there is relatively little truck traffic, short lengths of 8% grade on tangent are not objectionable, but long grades should not exceed 7% on tangent, and should be compensated on curves. A gently rolling profile with very few, if any, tangents produces a more pleasing appearance than a series of tangents connected with comparatively short vertical curves. At summits, vertical curves should be flat enough to provide a sight distance equal to the distance required to stop a vehicle. If the design is a two-lane, two-way road, then passing would not be permitted on summits, but would be provided for at frequent intervals by sections of road allowing much longer sight distances on horizontal and vertical curves.

The normal length of head-light beam will not be affected if vertical curves of the foregoing standard are used, because, especially on a black road, the lights are of little value more than 300 ft ahead of the car, or, say, 400 ft with the brightest lights allowed by present regulations. At that distance, the edge of the road cannot be distinguished. An object, such as a cow, in the

middle of the road is not recognized as an obstacle until the car is within 400 ft of it.

Sight Distance.—Tests conducted by the writer show that a wet bituminous surface may have a coefficient of friction as low as 0.54, and that when traveling at high speeds, it is dangerous to apply the brakes hard enough to develop all the coefficient of friction, especially on a curve. Using 0.40 as a safe maximum, and allowing 1.5 sec for the average driver to re-act in an emergency, the formula for stopping distance (on level grade) becomes,

$$s = 0.0835\ V^2 + 2.2\ V \qquad (23)$$

Sight distances, both on vertical and on horizontal curves, should be not less than the stopping distance, as computed, using the safe maximum speed in Equation (23).

Conclusion.—The writer endorses the remainder of the paper. Classifying accidents according to three types of contributing factors—faulty cars, roads, and drivers—only 5% of the accidents are found to be the result of faulty cars; about 20% result from faulty road construction, and the remaining three-fourths result from faulty drivers. It is true that there are often poor features of road design involved, but the drivers show bad judgment in not allowing for those poor features. In support of the foregoing, statistics show that, in any one year, about 75% of all drivers have no accidents, and that 5% of all drivers have about 30% of the accidents. Several commercial firms, employing more than 1 000 drivers, have succeeded in reducing their accident rate to one-fifth, by eliminating drivers who were frequent repeaters.

A study of railroad grade-crossing accidents provides some insight into the actions of the average driver. In several million observations, about 10% of all drivers were classified as not exercising reasonable care in crossing tracks. When drivers survive an accident, they often claim that they never saw a train on that particular crossing before, and, further, claim they neither heard nor saw the train, although facts prove that it was impossible for a normal person not to have done so. Perhaps that is why many drivers overtake and pass cars at summits of hills, because they have never been caught by a car coming toward them on that particular hill: No wonder there are so many collisions on the highways if some persons drive "blind" and "deaf", and if 10% do not exercise reasonable care in driving!

The writer recommends that about 5% of drivers, the more prolific accident-makers, be deprived of the privilege of using the highways. Then, with the improvement of highways by designing for a safe, reasonable speed, and the positive regulation of traffic to conform to that speed, the accident rate should be reduced to a point well below one-fifth the present rate. This can be done, as proved by Evanston, Ill., that cut its accident rate to only 7% of its former rate within a few years.

It is to be hoped that there soon will be a nation-wide standardization and enforcement of traffic rules and licensing of drivers, and a standardization of features of road design.

H. W. GIFFIN,[13] ASSOC M. AM. SOC. C. E. (by letter).[13a]—Some interesting questions as to the future of highway systems in the United States is raised in this paper. The subject deserves the attention of highway designers, administrators, and the general public. Before consideration of details of design three questions of a fundamental nature arise: (1) Is there a present need and demand for express highways as described in the paper? (2) Can they be operated safely? (3) Is there economic justification for them? Discussion of some of the phases of the present highway situation may throw some light on the future. Highway safety is much in the public mind and certainly every one concerned with the administration and design of highways should give serious thought to it in contemplating the highways of the future. A favorite method of foretelling the future is to examine the past for trends and to project these trends forward. Trends have a way of continuing, but many of them decrease in pace and some, finally, come to an end. Each trend should be examined carefully for those factors responsible for its growth and the conditions favoring its continuance.

One of the conspicuous trends of the past is the higher average speed of automobile operation. Four-wheel brakes, low centers of gravity, better tires, better metals in improved power plants, and straighter, wider, and better surfaced highways have supported this trend. Will it continue and, if so, how far? Will it reach 100 miles per hr as an average, or for large numbers of vehicles, on the main roads? To-day, occasional vehicles operate at that speed at a few places. Is this an indication of future average conditions? On the other hand, what tends to arrest this trend?

Perhaps the increase in the accident rate which is now arousing the public will act as a damper. Can accidents be attributed to speed? Perhaps not altogether, but few will deny that speed is a contributing cause. Of ten accidents picked at random, how many would have been avoided by a reasonable reduction in speed? The effect of a reduction in speed is to give a greater margin of time and distance to avoid trouble. As most accidents occur within 2 or 3 sec, an extra second means much in deceleration or in steering clear of an object. Furthermore, the seriousness of many accidents is a function of the speed at contact.

It is maintained by some that the combination of speed and bad driving, and not speed in itself, is the cause of accidents. Most drivers over-estimate their ability to drive safely at the higher speeds. The formation of situations likely to result in accidents is of very frequent occurrence. Often, considerable skill is necessary to avoid contact. Should not good driving prevent the formation of such situations? One might define good driving as always having the vehicle under that degree of control which permits a sufficient margin of time and distance to take appropriate action to avoid contact. By this definition how many drivers would be classed as good? Even the best of drivers have lapses of inattention and absent-mindedness, and are saved from accident by fortunate circumstances. Any interested observer can

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[13a] Received by the Secretary November 4, 1936.

testify to the many instances of bad driving he has seen. How many drivers never drive faster than is warranted by the range of their lights? What is the proportion of good and bad driving on high-speed highways? Should not something be done about the bad driving before average speeds increase?

Perhaps in the future there will be some method of eliminating the bad drivers and educating the others into being good drivers. How fast, then, can a good driver drive? The manufacturer constantly is improving the "road-ability" of the automobile so that the individual can keep the vehicle on its course more easily. No one can foretell what the limit may be. Probably, however, there will always be a need of sudden stops as long as the operation of automobiles remains subject to the whims and judgment of individuals. As far as can be foretold now, the rate of deceleration of the vehicle must always depend on friction. The surfaces of roads, brakes, and tires may be improved in this respect, but it does not appear that any great improvement can be made in this direction. Certainly, the rate of vehicle deceleration cannot go beyond the possible rate of deceleration of the human body without injury.

With all the engineer can do to build safety into the road he cannot build good driving by individuals into it. That must always remain a matter of individual responsibility. Whether or not the engineer improves alignment, widens pavement and shoulders, separates grades by bridges, separates opposing lines of traffic, and solves the problem of posting adequate and proper signs, there will always be individuals who will drive carelessly and too fast for the conditions. There are some hazards, furthermore, which the engineer cannot remove, such as fog, sleet, and blinding snowstorms. It lies within the ability of the highway designer to design reasonably safe roads for the use of prudent drivers. Is it possible or desirable to try to go beyond this ideal? Many modern roads have all the features of safe design for reasonably prudent drivers, and yet the accident toll on them continues at an alarming rate. It seems that the removal of each hazard adds an increment of speed.

From an economic standpoint the justification of the construction of express highways must rest on their capacity to carry a very large number of vehicles and to save for them something worth the cost of the highway. Either distance, time, annoyance, operating cost, or dangerous conditions must be decreased. Distance often can be reduced by re-alignment, but for the main lines of travel it is doubtful whether this reduction would amount to more than 5% were all the main roads rebuilt on new alignment. Saving in time would depend on the saving in distance, elimination of stops, and a higher rate of speeed. The operating cost of automobiles is a function of speed, the increase being very rapid beyond 40 miles per hr. Improvements in the vehicle of the future will undoubtedly effect greater savings in operating cost, but the relationship of increased cost for high speeds will probably hold. Hazard on the main roads is almost entirely a function of speed and of the volume of traffic.

There is another economic aspect to the subject of express highways. There are now extensive systems of transportation by road, rail, water, and air in the United States. Each system is capable of producing a certain kind of service with the greatest advantage. Some overlapping of service is inevitable, and even essential, so that each system will render the best possible service in its field. The modern highway has already encroached upon the field formerly regarded as belonging to the railroad, forcing the railroad companies to re-adjust the service offered to the public. This development can be regarded as healthy as far as it has gone to-day and perhaps it can go still further without harm; but it seems probable that the principal railroads of the future will furnish a service not economically furnished by the highways. Is there need and economic justification of providing complete duplication? Are there not many highways yet to be built before there is necessity for further rapid encroachment on the railroad field? The railroads are adjusting their service to provide safe and comfortable means of travel for long distance at high speed. For speeds beyond those which the railroads of the future will provide there are airplane systems. Is the highway, with its individualistic mode of operation, an ideal system for providing all kinds of transportation service?

As better highways are built, the older roads suffer by comparison in the public mind; but all roads not conforming to the highest standards of design should not be thought obsolete. Their useful life is not nearly ended because they must be an important part of the highway system for many years. Day after day many of them are providing a service commensurate with the needs, and producing a return commensurate with the investment in them. Of course, many others can be rebuilt economically and, when rebuilt, should be changed to provide better service; but there are many which, although they are not so efficient as new ones would be, cannot be re-designed for a long time. Nor are they generally regarded as dangerous, inasmuch as they are driven at slower and more economical speeds by those who have become accustomed to them. An examination of the accident records of these roads does not indicate that they should be discarded.

There is one accident for approximately 200 000 miles of driving. If one could witness 200 000 miles of driving by one person—or better, 50 miles by 4 000 persons—over average streets and roads, one would see a wide range of speeds, great difference in the skilful handling of motor vehicles, and all shades of driving between good and bad; and yet only one accident would occur involving injury to driver, passenger, or pedestrian; 3 999 out of 4 000 would come through a 50-mile trip safely. What is a reasonable conclusion to draw from this fact?

Although the automobile manufacturer has done a good job in production, both from an economic standpoint and from that of convenience and safety, modern automobiles are now equipped with power plants far beyond the needs or even desires of the average motorist. This excess power is seldom used; it is an economic waste and probably a contributing cause for much bad driving. It has been said that excess power is very convenient in an emergency.

On the other hand, the mere possession of it often invites the creation of such an emergency. The lack of it would force a reduction in high speed and, by and large, would be far safer. It is emergencies of this nature which require exceptionally good driving by others on the road. Is the fact that the manufacturer is providing something not needed and not in the public interest a valid reason for building roads to keep pace with it? If permitted, the manufacturer of the future will probably be able to supply power plants capable of speeds of 200 or even 300 miles per hr. Imperfections of the mechanical machine are gradually being overcome; but the human machine is the limiting factor in the attainment of highway safety.

The problem of the future highway, then, is related to the capacity of individuals to drive safely, to their economic needs, and to the standard of living of the motorist on which the cost of improvement must rest. There is much that is not clear, much that is unknown, and many questions to be answered before the specification for the future highway can be written. In the meantime, the highway engineer will continue to do what he can to interpret the immediate needs of the large majority of motorists, to build well in accordance with the funds provided, and to try to eliminate road hazards which conceivably are contributing causes of accidents. What lies beyond must await future developments.

T. T. WILEY,[14] JUN. AM. SOC. C. E. (by letter).[14a]—In presenting this paper the author has afforded a timely opportunity for discussing some of the factors that affect the design of the modern highway. The design of many highways and highway structures under construction to-day is scarcely adequate for safe driving at modern-vehicle road speeds and, if the present progress in motor-vehicle design is continued, the roads being built to-day will be obsolete almost before they are completed. A discussion of superelevation and curves seems especially pertinent, because these are features of design that have a direct bearing on safety and that have not been given the attention and study that they deserve.

It is true that Equation (1) yields rates of superelevation that are higher than required for safe and comfortable riding. However, the practice of using a speed lower than the design speed in this equation is not satisfactory. A more scientific attack on this problem can be made by considering the forces that prevent side-slip outward when the speed of travel is such that the resultant of the mass of the vehicle and centrifugal force is not normal to the pavement. When these frictional forces are considered, Equation (1) becomes,

$$E = 0.067 \frac{V^2}{r} - f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

in which f is the average coefficient of friction developed between the tires and the pavement in order to prevent side-slip.

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[14a] Received by the Secretary November 5, 1936.

Values of f varying from 0.15 to 0.20 have been used, but investigations have indicated that they are too high for comfortable riding and safe driving. The writer conducted a brief series of tests to determine the safe speed on curves, using Equation (24) and the definition that the safe speed was "the minimum speed at which the driver or passenger feels a side pitch outward." Two cars with a driver and one or two passengers were used for this investigation. One car was equipped with knee action and the other with transverse springs.

Test runs were made in both directions on seventeen curves, and the speeds at which side pitch became noticeable were determined Additional runs were made at speeds calculated from values of $f = 0.15$ and $f = 0.10$. Table 3 gives the curve data, test speeds, and values of f.

TABLE 3.—TESTS TO DETERMINE SAFE SPEED ON CURVES

Curve No.	Degree	Superelevation, in feet per foot	Speed for side pitch, in miles per hour	Coefficient of friction, f, for side pitch	Velocity, V, in Miles per Hour: For $f = 0.15$	Velocity, V, in Miles per Hour: For $f = 0.10$	Curve No.	Degree	Superelevation, in feet per foot	Speed for side pitch, in miles per hour	Coefficient of friction, f, for side pitch	Velocity, V, in Miles per Hour: For $f = 0.15$	Velocity, V, in Miles per Hour: For $f = 0.10$
1	11	0.05	35	0.11	39	34	9	3	0.058	60	0.07	77	67
2	17	0.062	30	0.12	33	28	10	5	0.092	50	0.06	64	57
3	11	0.075	30	0.04	42	37	11	5.5	0.067	45	0.06	59	51
4	10	0.046	30	0.06	41	35	12	4	0.042	50	0.08	64	55
5	6	0.025	35	0.06	50	42	13	24	0.083	25	0.10	29	25
6	5	0.017	35	0.07	53	45	14	5.5	0.083	45	0.05	60	53
7	9	0.042	33	0.07	41	37	15	5.5	0.083	45	0.05	60	53
8	3	0.000	50	0.09	65	53	16	13	0.054	30	0.06	36	32
							17	12	0.062	30	0.07	39	34

The following conclusions were obtained as the unanimous opinion of all persons who participated in the tests:

(1) The speeds at which side pitch first becomes noticeable are slower than necessary for comfort or safety;

(2) The speeds which require a friction coefficient of 0.15 create very definite side pitches of such magnitude as to be uncomfortable, require undue effort in steering, and often border on being dangerous, particularly when meeting traffic; and,

(3) The speeds which develop a friction coefficient of 0.10 create definite side pitches that are reasonably comfortable, require appreciable but not excessive steering effort, are not dangerous, and are fast and reasonable speeds in every case.

In the author's example for determining the minimum radius by using Equation (1), the design speed was 100 miles per hr and the rate of superelevation was 1 in. per ft of width. Then, the assumption was made that it would be satisfactory to design for 60 miles per hr and the minimum radius was calculated as 2 900 ft. The danger in making such an assumption is

shown by taking the same example and solving for the radius, using the true design speed of 100 miles per hr and Equation (24) in which f is taken as 0.1. From this formula, the calculated minimum radius becomes 3 700 ft. The difference between this radius and the one obtained by using the methods now in common use certainly indicates the need for more scientific design of curves, particularly for high speeds.

Table 3 shows a considerable range in the values of f for speeds which first create side-pitch. There are several reasons for this, one of which is the personal element which existed in noting side-pitch. However, certain physical characteristics of the curves also cause these differences. Such items as grade, length of run-off, and roughness of the pavement, undoubtedly affect the results of such tests and indicate that these items must be considered in design. This leads directly to the author's discussion of the transition between normal crown and superelevation, and his statement that "it is doubtful whether spiral curves will be necessary."

If the design is "to be such that the vehicle may enter and leave the curve smoothly—almost automatically", the spiral will be necessary on all superelevated curves. Although engineers know that the spiral provides a uniform rate of change in degree of curve, many of them fail to realize that its principal advantage is that it also provides a uniform rate of change in the superelevation so that the superelevation at any point on the spiral or curve is in exact agreement with the degree of curve at that point.

There is no need for presenting the theory of the spiral because several useful spirals have been developed. However, railroad engineers have learned considerable about curves and spirals, and their knowledge can, and should, be applied to highway design; for example, they have found that superelevation cannot be introduced across the track gage at a rate greater than $1\frac{1}{4}$ in. per sec without creating an uncomfortable riding condition. Therefore, superelevation on a highway should not be introduced at a rate exceeding 0.02 ft per ft per sec, and the length of spiral on a curve with a superelevation of 1 in. per ft should approximate the distance traveled in 4 sec at the design speed. On this basis, curves designed for a speed of 100 miles per hr, with superelevation of 1 in. per ft, should have a spiral approximately 600 ft long.

Highway engineers are loath to accept the spiral as a necessary feature of design, partly because they do not realize the ease with which it can be calculated and applied. The writer was taught a type of spiral which requires about 10 min of calculation in preparing the field notes and a negligible amount of excess time in staking out the spiral and curve. Resort to tables is unnecessary.

The author states that "failure to attribute accidents to faulty road design is due to several causes principal of which is the subtlety of the problem." This is probably true especially with respect to accidents at curves, which too often are listed as being caused by driving too fast for conditions.

An automobile entering a curve is subjected to new forces and, at modern fast speeds, these forces often change in magnitude and direction very rapidly. Superelevation for curves is usually introduced on the tangent over a dis-

tance termed the "run-off." A vehicle traveling over the run-off is subjected to a lateral force which must be overcome by turning the steering wheel in a direction opposite to that of the curve. Otherwise, the vehicle will move toward the inside of the curve, thus crowding the center line or pavement edge with occasional disastrous results.

The instant the car enters the curve, forces acting on the car change instantaneously, requiring a very rapid and usually complete reversal in the direction of steering. The changes may be so great that the steering wheel cannot be turned rapidly enough and the car is thrown partly or wholly out of control and an accident results. The subtlety of the problem now enters, for the accident report states "driving too fast for conditions." Is the mistake in the driving or in the conditions? The common assumption is that it is the driving. Engineers familiar with the spiral will claim that the conditions are at fault. There is absolutely no reason why the driver should have to attempt to overcome the sudden change in forces because a properly designed curve will be spiraled and a sudden change of any appreciable magnitude in the forces which must be overcome by steering will never occur.

The writer has expressed the opinion that Equation (24) gives satisfactory results in computing curve requirements. This formula was not advocated because it is better than Equation (1), but because it is better than Equation (1) as ordinarily used. In other words, Equation (24) is a logical way to take into consideration the degree of overbalancing that can be tolerated in rounding a curve, whereas the attempt to do this by substituting a speed less than the design speed in Equation (1) is not based upon scientific analysis and is often unsatisfactory. Equation (24) should afford a desirable "minimum standard", but if highways are to be designed for the greatest safety, why not adopt the best available, which is Equation (1) using the true design speed rather than a guess at a "safe" speed? Similarly, spiraled curves are certain to be safer than non-spiraled ones. Use the spiral.

Incidentally, this contention is applicable to many other design standards. Too often a "minimum standard" becomes "the" standard, so that, in many instances, standard curves of 1 500-ft radius have been built where flatter curves would have been just as economical and certainly more desirable. The highway engineer must remember that a minimum standard is the worst that he is permitted to use, and that good design requires the use of something better than the minimum allowable if it is reasonably practical. To use minimum standards when better can be obtained is usually "penny wise and accident foolish."

The writer cannot agree with the author's general contention that signs constitute a hazard. Wooden posts, 4 in. square, furnish excellent mounting for signs and, if struck by a car moving at a fair rate of speed, they snap off without causing appreciable damage to the car. The types of signs now in use would be useless because of lack of visibility, if suspended from overhead cables. Flexible standards of sufficient rigidity to furnish adequate support for standard signs subjected to wind pressure, vandalism, and other buffetings would be just as dangerous as wooden posts.

A properly designed modern highway should require few signs. If the design speed is to be placed at a limit, such as 100 miles per hr, the "standard" signs now advocated will be wofully inadequate.

The author is to be commended for emphasizing the fact that the highways being built to-day are scarcely abreast of the car of to-day, not to mention the car of the future. The highway engineer must exercise his imagination to visualize the conditions that will exist in the years to come, and base his work upon the conviction that the best design is scarcely good enough.

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DISCUSSIONS

SIMPLIFIED METHOD OF DETERMINING TRUE BEARINGS OF A LINE

Discussion

BY MESSRS. O. H. CHILTON, CHALMERS C. SCHRONTZ, FRANK M. JOHNSON, WALTER H. STARKWEATHER, AND C. I. DAY.

O. H. CHILTON,[12] ESQ. (by letter).[12a]—Many engineers, to whom the intricacies of field astronomy are a mystery, will welcome this paper and the simplification of the work of finding the azimuth of a line which it enables. For the very reason that it is addressed to those who are not fully acquainted with the subject, it seems all the more desirable to draw attention to some points which, as they stand at present, are likely to cause confusion.

Consideration of the formulas of the paper shows that the exact time of observation does not enter into the computations. The only purpose for which the time is observed is that of obtaining the correct declination of the sun from the ephemeris.

The limitations of Table 1 in its present form with somewhat wide intervals of 1°, will result in no great accuracy in the azimuth obtained. The author suggests a probable error of 1′. Accepting some such degree of accuracy, it can be easily shown that even at the more unfavorable seasons of the year, when the sun's declination is changing most rapidly, if the time of observation is known to the nearest 10 min, the present purpose will be adequately served. Thus, it would be quite sufficient for the observer to look at his watch at the beginning and at the end of the observations, and to take the mean of the times, or even to record merely one time, to the nearest minute, when reversing the instrument between pointings at the sun. The recording and averaging of times to the nearest 5 sec in Table 2 is not necessary and is misleading.

In the instructions for the field work there should be a clear statement that observations for azimuth should not be made on the sun except when it is favorably situated in the heavens. It can be shown that the effects,

NOTE.—The paper by Philip L. Inch, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Earl F. Church, Paul E. Wylie, James B. Goodwin, C. H. Swick, Philip Kissam, and George D. Whitmore.

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[12a] Received by the Secretary, October 20, 1936.

on the azimuth obtained, of errors in the assumed values of the latitude and the declination, are least when the sun is on the prime vertical—that is, due east and west. However, for the present purpose, it would suffice to limit observations to those periods when the sun is more than 3 hr off the meridian, that is, to mornings before 9:00 A. M., and afternoons after 3:00 P. M.—always having regard to the possible incidence of summer (daylight saving) time and any large variation between the local civil time and the true sun time at the place of observation.

The engineer will require some indication of the accuracy requisite in the value of the latitude of the place of observation, which he is instructed to obtain from "an accurate map." A map may be accurate, but the scale may be too small for some purposes. By differentiation of the fundamental equation relating the azimuth to the observed and assumed quantities it can be shown that:

$$\delta Z'' = -\delta c'' \frac{\cot t}{\sin c} \qquad (8)$$

in which Z = azimuth; $c = 90° - \phi$; and t = hour angle.

Assuming that azimuth observations will be made by this method only when the sun is more than 3 hr off the meridian, and that the paper is addressed to engineers working in the United States between limits of latitude of, say, 49° and 30° N, it can easily be computed that for $\delta Z = 60''$, δc has a mean value of about 45″. There will be other sources of error in the computation, although possibly operating to balance each other. However, allowing, say, one-third of the error to this source (say, 15″ in the latitude, which is about 1 500 ft on the ground) a map of not less than 0.5 in. to 1 mile is indicated.

It is unfortunate that the author has chosen an example solved as long ago as 1910, and has included in the text a statement which was true for that year, and indeed until 1924. The American Ephemeris, in common with the ephemerides of other countries such as the British "Nautical Almanac", now gives the elements of the sun's position, such as the declination, for each day "for 0 hours Greenwich Civil Time which is 12 hours before Greenwich Mean Noon of same date." A mean hourly variation of declination is no longer tabulated, but finite differences between each of the daily values are given. As it stands at present the text would cause much confusion to an engineer, uninstructed in these matters, who was attempting to apply the method.

With regard to a possible extension of Table 1 the proposed range of values of the vertical angle and of the latitude, is not stated. In Table 1 as printed[12b], an altitude as low as 21° is given. This might be open to question on account of the magnitude of the refraction correction involved. A lower limit of 25° is suggested. If it is intended to compute the table for latitudes north of the United States, the unsuitability of the method to very high latitudes will doubtless be borne in mind.

[12b] Correction for *Transactions:* In Table 1, Column (20), for $h = 32°$ (A), change 42.1 to 42.7.

Until the limits of latitude for which the table is to be computed are known, it is not possible to comment finally on its form, but even for the part of the table printed, there is doubt as to its suitability. The writer has compared the methods of computation of the worked example, and the azimuth so obtained appears to differ by 45″ from that given by Table 1. Taking the author's standard of accuracy, "a probable error of 1′", a discrepancy of 45″ arising solely from the method of interpolation from the table is too great to be acceptable when there are other sources of error all operating to effect the final accuracy of the determination, such as errors in the assumed latitude of the place, sun's declination, and the observed angles.

Examination of the part of the table available shows that the method of linear interpolation by first differences becomes still more inadequate as the values of altitude and latitude increase. The worked example is taken from the most favorable part of the table. As it stands it could, of course, be made to serve its purpose if interpolation by second differences is resorted to; but that is scarcely likely to appeal to the engineer in the field, and indeed a direct computation by formula adapted to logarithmic working would be almost as rapid. If the form of the table is to be retained with interpolation direct by first differences, then it does appear that the intervals of 1° in both altitude and latitude may have to be broken down to 30′, or perhaps 20′, for the higher values.

CHALMERS C. SCHRONTZ,[13] M. AM. SOC. C. E. (by letter).[13a]—Equation (1) of this paper can be solved mechanically by the solar compass or by a solar attachment to the transit when the apparent altitude and declination of the sun are employed instead of their true values, as used in the method by direct observation. There are no means by which the correction for direct refraction in altitude can be made in the use of the solar attachment. A compensating correction must be made which can be applied to the declination arc of the solar attachment.

The declination correction is applied to an angle normal to the plane of the equator, whereas the direct refraction is applied to an angle normal to the horizon at the place of observation. Consequently, the declination correction for refraction is an increment of an angle in a system of polar co-ordinates corresponding to an increment of an angle in another system of polar co-ordinates, the two systems of which are coincident in the vertical plane passing through the pole of one, and the zenith of the other system at the place of observation. Therefore, the refraction correction for declination is mathematically comparable with the direct refraction for altitude.

This relation of polar co-ordinates has made it possible to solve this problem mechanically by the use of the solar attachment and the same solution may be made mathematically from a direct observation of the sun.

The table for corrections to be applied to the true declination in order to determine the apparent declination of the sun for the hour and latitude of the

[13] Associate Engr., U. S. Eng. Dept., Jacksonville, Fla.

[13a] Received by the Secretary October 22, 1936.

place always accompanies the solar ephemeris. In either case, whether the solution of the spherical triangle is to be made from the true or from the apparent altitude of the sun, the declination as given in the ephemeris for noon at Greenwich must be corrected for the hourly change from Greenwich noon to the time of observation. The apparent declination is then obtained by the addition or subtraction of the refraction correction for declination corresponding to the time and latitude of the place of observation. When the apparent declination is used, these two corrections for hourly change and refraction are the only ones required, no correction being necessary for the observed altitude of the sun.

In the writer's experience this problem can be solved most satisfactorily by the use of a table of logarithmic and natural trigonometric functions, together with a solar ephemeris (furnished gratis by the instrument manufacturers), and a map from which the longitude and latitude of the place can be determined. In the tables of logarithmic functions the cosines and tangents of the latitudes occur on the same page and on the same line and also the same functions, for the vertical angles, which greatly facilitates the selection of the values of functions. Examples of the solution of the formula based upon the true and apparent altitude and declination follow.

Example 2.—Solution based upon apparent altitude and declination of the sun:

Date: April 30, 1936.
Place: Latitude, 40° North; longitude, 105° West.
Declination sun, Greenwich, noon, N 14° 48.3′.
Hourly change, + 0.76′ (declination increasing).
Time of observation: 9:00 A. M. (local mean time).
Altitude of sun's center, 43° 31′ 30″. (No correction to be applied to observed altitude.)

Apparent declination of sun at time of observation:

Longitude, 105° West, at 15° per hr is 7 hr from Greenwich, noon, to place of observation, noon.
Time of observation, 9:00 A. M., is 3 hr before noon at place of observation.
∴ Time of observation is 7 − 3 = 4 hr after noon at Greenwich.

Declination, Greenwich, noon............		N. 14° 48′ 18″
Hourly change, 4 × 0.76′..................		0° 3′ 2″
Declination at time of observation........		14° 51′ 20″
Refraction for third hour (from noon), latitude 40° N........................	+	0° 0′ 40″
Apparent declination for time and place...		14° 52′ 00″

Solution:

$$\phi = 40^\circ \quad h = 43^\circ\ 31'\ 30''; \text{ and, } \delta = +\ 14^\circ\ 52'\ 00''$$

Logarithms:

cos ϕ..........	9.884254	tan ϕ..........	9.923713
	9.860382	tan h..........	9.977630
log.............	9.744636	log.............	9.901443
Co-log............	10.255364		
sin δ..........	9.409207		
log.............	9.664571		

Natural functions (corresponding to foregoing logarithms):

(+) 0.461924 (−) 0.796971

(−) 0.796971

cos Z = (−) 0.335047; and, Z = 70° 25′ 49″

Example 3.—Computation based upon true altitude and declination of the sun:

Observed altitude, sun's center..............	43° 31′ 30″
Correction for refraction and earth's parallax, 0.9′	− 0° 0′ 54″
True altitude	43° 30′ 26″
Sun's true declination for time of observation (as in Example 2)......................	14° 51′ 20″

Solution:

$$\phi = 40°;\ h = 43°\ 30'\ 26'';\ \text{and},\ \delta = 14°\ 51'\ 20''$$

Logarithms:

cos ϕ..........	9.884254	tan ϕ..........	9.923813
cos h..........	9.860510	tan h..........	9.977360
log.............	9.744764	log.............	9.901173
Co-log............	10.255236		
sin δ..........	9.408890		
log.............	9.664126		

Natural functions (corresponding to foregoing logarithms):

(+) 0.461451

(−) 0.796475

cos Z = (−) 0.335024; and, Z = 70° 25′ 35″

The difference in the two results, of 14″ of arc, is deemed negligible in view of the interpolation of the values in the tables of functions for seconds of arc and the tabulated difference in refraction for the two systems of polar co-ordinates.

The signs of the declination (+ for north and − for south) become important when the direction of the sun is close to the east or west point from the point of observation. If the result, $\cos Z$, is +, the direction of the sun is to be referred to the north point, and when it is −, as in this case, the direction is to be referred to the south point.

It is to be noted that the $\cos \phi$, $\tan \phi$, and $\sin \delta$ can be taken as constants for several observations taken within 15 or 20 min without appreciable error in the results. This provides ample time to make several observations in which only the altitude and azimuth of the sun will change materially. The arrangement of the solution lends itself to several computations simply by supplying the changed values of $\cos h$ and $\tan h$. Consequently, the solution is thus expedited.

Field Operations.—The travel of the sun's disk when viewed through the telescope of the solar attachment is parallel with the two horizontal cross-hairs and when the sun's disk is brought between these parallel cross-hairs by the lower tangent screw of the transit, the transit telescope points to the true north or south, provided, of course, that the proper latitude and apparent declination have been used. In this position it will be some time before the sun's disk can be detected to move above or below the horizontal cross-hairs of the solar telescope, which is due only to the sun's change in declination. This allows plenty of time to make a precise setting of the cross-hairs of the solar telescope on the limbs of the sun, and, hence, a precise pointing of the transit telescope on the true meridian. When the proper declination is once set off for the time and latitude of the place, one single observation is sufficient to determine the meridian.

In a direct solar observation the sun does not travel parallel with either the horizontal or the vertical cross-hairs of the telescope, but in a diagonal direction across the field of vision. In the usual method of pointing the telescope for forenoon observations, the vertical and horizontal cross-hairs are brought simultaneously in contact with the right and upper limbs of the sun, recording the vertical and azimuth angles; then reversing the telescope and setting the cross-hairs in contact with the left and lower limbs of the sun and recording the vertical and azimuth angles. For afternoon observations, the lower and right, and upper and left, limbs are observed and recorded in the same way. The means of the vertical and azimuth angles for each pair of observations are taken as the apparent altitude and true azimuth of the sun's center for the mean time of the two observations.

It is evident that such observations cannot be made with the ease and precision attained in the setting with the solar attachment. The results obtained in a direct observation are never so uniform, satisfactory, or trustworthy as those obtained with the solar attachment, unless in the case of two or more direct solar observations. One single determination from direct observation is scarcely trustworthy.

The results of a direct observation are easier, quicker, more uniform, and more satisfactory when the observation is made upon the center and tangent to the lower limb of the sun by bisecting, with the vertical cross-hair, a small

segment of the sun just before contact of the horizontal cross-hair with the lower limb of the sun. To the vertical angle thus obtained the sun's mean semi-diameter is added, 16′ of arc, to obtain the altitude of the sun's center. The greatest variation of the sun's mean semi-diameter is 18″ of arc, and may be neglected unless greater precision is required. The horizontal angle is the azimuth of the sun, and no correction to it is necessary. Five observations are usually taken alternatively in direct and reverse positions of the transit. Each observation is treated independently. At least two observations are used to determine the azimuth. When these agree within less than 1′ of arc the mean of the two results is accepted. If not, one or more of the remaining observations are used for azimuth determinations. Seldom more than two or three determinations are necessary to obtain the desired result. The apparent declination is determined for the mean time of all the observations and is used as a constant for all the observations, so that for each independent determination of azimuth of the sun there are three constant values that must be taken once only from the tables of trigonometric functions, namely, cos ϕ, tan ϕ, and sin δ.

In this procedure one erratic result occasionally may be obtained in the azimuth determination which will evidence an error either in the solution or in the observation and may be corrected or discarded. The error thus discovered is confined to one single observation, whereas in the use of the mean of two observations, the error is reduced one-half and usually is not discoverable.

By the use of these expedients and the arrangement for the solution of the formula for azimuth, a set of observationes and computations may be made in the field with little delay, and the only tables required are the regular trigonometric tables which are familiar to all engineers and surveyors. Additional tables seem scarcely necessary and do not materially simplify the solution of the formula for azimuth.

The factor, A, is the natural number corresponding to the co-logarithm of cos ϕ cos h; or if natural functions are used, it is the reciprocal of cos ϕ cos h in which, for either case, the correction for refraction and parallax has been applied to h.

The factor B, is the natural number corresponding to tan ϕ tan h, also in which the correction for refraction and parallax has been applied to h; but if the apparent altitude of the sun is used there is no correction to be applied to h and all the correction for refraction and parallax is applied only once to one value of the declination, which may be used for several azimuth determinations, provided not more than 10 to 20 min are consumed in making the observations.

It is not the writer's purpose to criticize or to discourage any such worthy purpose or effort to simplify the solution of the azimuth formula. It is only to call attention to the fact that the apparent altitude of the sun may be used in solving the azimuth formula, and by so doing the refraction correction is referred and applied to the declination, once and for all, for a single set of several observations, and that the solution of the formula is made easily

and quickly by a simple arrangement and use of a logarithmic function already familiar to the engineer and the surveyor.

The writer is unaware of the use of the apparent altitude and declination having been suggested heretofore, or used, in the determination of the true meridian by direct solar observation. It has only been suggested to him by the fact that in the use of a solar transit the solution of this formula is accomplished mechanically. It would seem that the mathematical solution has been overlooked.

It has always been somewhat of a mystery why the solar attachment has not come into more general use. The attitude seems to be that its use is derogatory to the dignity of the engineer, or an admission of a lack of scientific technique and ability, and that the transit and its use is the acme of perfection. Except for those who are continuously employed in public land surveys, the expense of the solar attachment seems a burden. Its occasional use does not justify the expense. Many are not fully convinced of its accuracy and dependability. Its adjustments are thought to be difficult and can be made satisfactorily only by the instrument makers. Furthermore, it adds weight and makes the transit unbalanced, or otherwise interferes with the routine work of the transit. All these objections are largely imaginary, and a very little experience in its use will dissipate such objections.

FRANK M. JOHNSON,[14] M. AM. SOC. C. E. (by letter).[14a]—The method proposed in this paper has been studied carefully to ascertain whether it does, in fact, afford a saving in the number of steps ordinarily required in the use of trigonometric tables. The saving, if any, is not as great as it is made to appear by the author.

It seems better to state frankly that there are no so-called "short-cuts" in making direct observations on the sun for azimuth, or in the reduction of steps by the trigonometric process, if reliable results are to be secured. By "reliable" is meant the ascertainment of results that can be verified within the limits of the accuracy of the instrument used for the observation; this should be less than 1′, and with care may be kept within 15″. Observations and results should be such as may be verified by others if occasion arises, thus rem ving elements of doubt or approximation.

This paper is not a satisfactory substitute for the explanations on the subject which may be found in textbooks on surveying, and several official publications. It is unfortunate to create the impression that the textbook treatment of the subject has been made difficult; the method has been brought into general use, and the number of examples offered are designed to clear all questions of doubt if the engineer will make a fair attempt to grasp the several steps and make a few practice observations.

The use of the tabulations in the paper does not dispense with the need for tables of natural sines and cosines, and it is not believed that their use materially lessens the time required to resolve the formula in the usual way.

[14] U. S. Superv. of Surveys, Dept. of the Interior, Denver, Colo.

[14a] Received by the Secretary October 23, 1936.

On the other hand, it is felt that the method adds materially to the uncertainties of the results in case of one or more errors in making the observations, or other discrepancies.

TABLE 4.—FIRST AND SECOND DIFFERENCES FOR FACTORS *A* AND *B*, TABLE 1

Vertical angle, *h*, in degrees	FACTOR *A*			FACTOR *B*		
	Factor*	First difference, Δ_1	Second difference, Δ_2	Factor†	First difference, Δ_1	Second difference, Δ_2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
21	1.34085			0.28866		
		927			1 522	
22	1.35012		53	0.30388		22
		980			1 544	
23	1.35992		56	0.31932		21
		1 036			1 565	
24	1.37028		58	0.33497		25
		1 094			1 590	
25	1.38122		61	0.35087		25
		1 155			1 615	
26	1.39277		63	0.36702		30
		1 218			1 645	
27	1.40495		64	0.38347		28
		1 282			1 673	
28	1.41777		67	0.40020		31
		1 349			1 704	
29	1.43126		72	0.41724		33
		1 421			1 737	
30	1.44547		73	0.43461		35
		1 494			1 772	
31	1.46041			0.45233		

* Values of Factor *A* from Column (3), Table 1. † Values of Factor *B* from Column (3), Table 1.

TABLE 5.—FIRST AND SECOND DIFFERENCES FOR FACTORS *A* AND *B*, TABLE 1

Latitude, ϕ, in degrees	FACTOR *A*			FACTOR *B*		
	Factor*	First difference, Δ_1	Second difference, Δ_2	Factor†	First difference, Δ_1	Second difference, Δ_2
(1)	(2)	(3)	(4)	(5)	(6)	(7)
37	1.52811			0.52723		
		2 060			1 941	
38	1.54871		105	0.54664		52
		2 165			1 993	
39	1.57036		111	0.56657		58
		2 276			2 051	
40	1.59312		116	0.58708		62
		2 392			2 113	
41	1.61704		125	0.60821		63
		2 517			2 176	
42	1.64221			0.62997		

* Values of Factor *A* for a vertical angle, $h = 35°$, in Table 1. † Values of Factor *B* for a vertical angle, $h = 35°$, in Table 1.

On examination, it is believed that the interval of Table 1[13b] is altogether too great, as demonstrated by Tables 4 and 5. Assuming that all these values have been carefully verified and are available as indicated, with an interval of 1° in latitude and 1° in vertical angle, it will be noted that a considerable number of pages similar to Table 4 will be required in order to provide for latitudes ranging from about 25 to 65°, and similar to Table 5 for vertical angles ranging from about 15 to 60 degrees. The third variable, that of the

[14b] Correction for *Transactions:* In Column (3) Table 1, change Factor *A*, for $h = 25°$ to "1.38122" instead of "1.38127".

sun's declination, as well as the final result in azimuth angle are provided for by the use of tables of natural sines and cosines.

The method of direct observation on the sun for azimuth is regarded in high favor by engineers, and has been used extensively for many years. Reliance upon compass meridians for all classes of public land surveys was discontinued in 1890. Federal surveying and mapping agencies have stressed the importance of exact methods so long that it seems out of place to suggest a practice that has the value of an approximation only.

An obvious improvement for the tabulation of values of Factors A and B, if the method is to be favored, is to make the intervals of the latitude and vertical angles not greater than 0° 10′, but this extends the tables to such a length that it is doubtful whether it would be worth while to go to that much trouble. The tabular differences will increase more rapidly in the higher latitudes, and as the vertical angles become greater, the results then become correspondingly more uncertain.

The example given by the author does not derive the value to the nearest even 1′, as the bearing of the line by trigonometric reduction i S 1° 00′ 02″ W (not S 1° 00′ 48″ W, as ascertained by the use of the factors, A and B).

The trigonometric steps are not many, as will be seen from the following:

Observed vertical angle		25° 25′ 30″
Refraction		− 0° 2′ 00″
Parallax		+ 0° 0′ 08″
True vertical angle		25° 23′ 38″
Latitude		38° 53′ 40″
Sun's declination, south		1° 02′ 16″
	log sin δ	8.257958
log cos ϕ	9.891149	
log cos h	9.955871	9.847020
	log	8.410938
(Sun, south declination)	nat (−)	0.02576
(−) log tan ϕ	9.906733	
log tan h	9.676423	
log	9.583156	
nat (−)	0.38296	0.38296
Sun's azimuth, nat cos, Z		0.40872
True bearing of sun		S 65° 52′ 32″ W
Angle from sun to flag		64° 52′ 30″
Bearing of line		S 1° 00′ 02″ W

WALTER H. STARKWEATHER,[15] M. AM. SOC. C. E. (by letter).[15a]—As arranged by Mr. Inch, with double increments for 1′ of altitude and 1′ of latitude, Table 1 is quite ingenious and should prove welcome to field engineers who

[15] Technical Asst. to Chf. Civ. Engr., U. S. Coast Guard Hdqrs., Washington, D. C.

[15a] Received by the Secretary, November 9, 1936.

have occasion to determine azimuths of lines, since it shortens the necessary computation considerably. One operation is also eliminated completely by having the factors in the table corrected for refraction in altitude.

There are several points which might prove to be confusing, however, to any one who had not used this formula or one evolved from it. For many years Equation (1) and formulas evolved from it have been used by field engineers and in the technical press with all the variables represented by letters of the English alphabet; but in most textbooks and handbooks, with a few exceptions, Greek letters have been used for some of the variables. The formula appears less formidable if stated as follows:

$$\cos Z = \frac{\sin d}{\cos h \cos L} - \tan h \tan L \ldots\ldots\ldots\ldots\ldots (9)$$

in which Z = the angle between the sun and the local meridian; d = the true declination of the sun; h = the altitude of the sun corrected for refraction; and L = the latitude of the observer.

TABLE 6.—NOTES FOR COMPUTING THE BEARINGS OF A LINE

d	$-1° 02.5'$	......
h	$25° 25.5'$	......
L	$38° 53.5'$	......
Factor A	For 25° h and 38° L	1.3999
Correction h'	$25.5' \times 1.9$	48
Correction L'	$53.5' \times 3.3$	177
A	$\frac{1}{\cos h \cos L}$	1.4226
Factor B	For 25° h and 38° h	0.3638
Correction h'	$25.5' \times 2.8$	71
Correction L'	$53.5' \times 2.2$	118
B	$\tan h \tan L$	-0.3827
sin. d	....	-0.0182
A sin. d	....	-0.0259
cos Z	$A \sin d - B$	-0.4086
Z	$65° 53'.1$	
Horizontal angle	$64° 52'.5$	......
Azimuth	$1° 01'$	......
Bearing	S 1° 01′ W	......

The writer wishes to present a plea for using d for declination and L for latitude, believing that some confusion will thereby be eliminated. As d and L are the initials of the words, declination and latitude, they would not be likely to cause the uncertainty as to what they represent as would the Greek letters, δ, ϕ, θ, and λ.

As the readings of the vertical angles usually cannot be taken closer than the nearest minute of arc with an ordinary transit, refining any of the computations to a point closer than 0.5′ of arc, or farther than four decimal places in the computations, is not justified, as the results within the required limits will be obtained without these refinements. For the same reason, the time is not needed to seconds, as the nearest 15′ to 20′ of the correct time will define the declination of the sun sufficiently close.

In Table 6 the computation is made from the values used for Example 1 of the paper, arranged for convenient placing on the page of a standard field

notebook, and is carried only to the limits suggested herein. The results are within less than 0.5′ of arc of the values calculated to seconds of arc with five-place computations. This is believed to be sufficiently close, as authorities are apparently agreed that the accuracy of the results by this method is limited to one-half the least reading on the horizontal or vertical arc, in making the observation, and the author writes "allowing a probable error of 1′." but the azimuth has been computed to seconds.

C. I. DAY,[16] ASSOC. M. AM. SOC. C. E. (by letter).[16a]—The avowed purpose of this method is to improve the accuracy of true bearing determinations, now largely dependent upon the compass needle. Many county surveyors and others who have not been fortunate enough to receive the necssary mathematical training avoid direct readings on the sun because of the complicated appearance of the formulas. This has been somewhat remedied by the various solar attachments that can be added to the transit, but which are too expensive for only occasional use.

The formula (Equation (1)) from which the author's tables (illustrated in part by Table 1) have been derived does not lend itself to rapid mathematical solution, particularly when logarithms are used. Therefore, the tables are faster to use and can be used by any surveyor who is not familiar with logarithms. The $\tan^2 \frac{1}{2} A$ formula lends itself to solution by use of logarithms in about the same time as the tables, unless a computing machine is used in making the cumbersome multiplications.

Some surveyors are always somewhat uncertain as to how to apply the refraction correction to altitude observations and for them the tables suggested by the author are a boon.

In comparing the tables against the formulas, the accuracy seems to be within that claimed and certainly is considerably better than can be gained with a compass. Mr. Inch is to be commended highly for his efforts.

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[16] Pres., W. & L. E. Gurley, Troy, N. Y.

[16a] Received by the Secretary November 16, 1936.

PROCEEDINGS

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A Quarterly Meeting will be held in New York, N. Y.

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January 20, 1937:

Morning.—Annual Meeting. Conferring of Honorary Membership, and Presentation of Medals and Prizes.

Afternoon.—General Society Meeting.

Evening.—President's and Honorary Members' Reception and Dance.

January 21, 1937:

Morning.—Technical Division Sessions.

Afternoon.—Technical Division Sessions.

Afternoon.—Entertainment for Ladies.

Evening.—Entertainment and Smoker.

January 22, 1937:

All-Day Excursion.

January 23, 1937:

Inspection Trips.

The Reading Room of the Society is open from 9:00 A.M. to 5:00 P.M. every day, except Saturdays when it is closed at 12:00 M. It is closed all day on Sundays and holidays.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is an ample file of current periodicals, the latest technical books, and the room is well supplied with writing tables.